Quiz 1 Solution

1. Determine whether the series $\sum_{n=1}^{\infty}(-1)^n \frac{n^3 5^n}{(2n)!}$ converges or diverges.

   Using the ratio test, we get the following

$$\begin{aligned}
\left|\frac{(-1)^{n+1}(n+1)^3 5^{n+1}}{(2(n+1))!}\cdot\frac{(2n)!}{(-1)n^3 5^n}\right| &= \left|\frac{(-1)(n+1)^3 5}{(2n+2)!}\cdot\frac{(2n)!}{n^3}\right| \\
&= \left|\frac{(-1)(n+1)^3 5}{(2n+2)(2n+1)(2n)!}\cdot\frac{(2n)!}{n^3}\right| \\
&= \left|\frac{(-1)(n+1)^3 5}{(2n+2)(2n+1)}\cdot\frac{1}{n^3}\right| \\
&= \left|\frac{5}{(2n+2)(2n+1)}\cdot\left(\frac{n+1}{n}\right)^3\right| \\
&\approx \frac{5n^3}{4n^5}\to 0 \qquad \text{as } n\to\infty
\end{aligned}$$

   Therefore the series is absolutely convergent by the ratio test.

2. Find the interval of convergence of the power series $\sum_{n=1}\infty\frac{(-3)^n}{\sqrt{n}}(x+2)^n$. Do not check the endpoints.

   In order to find the interval of convergence, we need to use the ratio test, and find the x values for which the series is absolutely convergent.

$$\begin{aligned}
\left|\frac{(-3)^{n+1}(x+2)^{n+1}}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{(-3)^n(x+2)^n}\right| &< 1 \\
\left|\frac{(-3)(x+2)}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{1}\right| &< 1 \\
\left|(-3)(x+2)\sqrt{\frac{n}{n+1}}\right| &< 1 \\
\left|(-3)(x+2)\right| &< 1 \qquad \text{as } n\to\infty \\
\left|(x+2)\right| &< \frac{1}{3} \\
-\frac{1}{3} < x+2 &< \frac{1}{3}
\end{aligned}$$

   Therefore the interval of convergence is $-\frac{7}{3} < x < -\frac{5}{3}$