AMATH 301
Homework 2: Autumn 2008

DUE: Thursday, October 23 at 3 a.m.

I Consider the circuit depicted in the figure. By using the two following facts:

- The voltage drop across a resistor is $V = IR$
- The sum of all voltage drops in a closed loop sum to zero

The currents I_1, I_2, and I_3 are determined from the 3×3 system.

$$\begin{aligned}
R_6 I_1 + R_1(I_1 - I_2) + R_2(I_1 - I_3) &= V_1 \\
R_3 I_2 + R_4(I_2 - I_3) + R_1(I_2 - I_1) &= V_2 \\
R_5 I_3 + R_4(I_3 - I_2) + R_2(I_3 - I_1) &= V_3
\end{aligned}$$

where $R_1 = 20$, $R_2 = 10$, $R_3 = 25$, $R_4 = 10$, $R_5 = 30$, $R_6 = 40$, $V_2 = 0$, $V_3 = 200$, and V_1 will be variable. In this form, the associated matrix is strictly diagonal dominant.

(a) Vary V_1 from 0 to 100 in steps of 2 (i.e. $V_1 = 0, 2, 4, ..., 100$) and calculate I_1, I_2 and I_3 as a function of increasing V_1 by solving the system with the backslash command. Save your results in a matrix of 3 columns and 51 rows where the first, second and third column are I_1, I_2 and I_3 respectively.

ANSWER: Should be written out as A1.dat

(b) Repeat part (a), but now solve it with three additional methods: **LU** decomposition, Jacobi Iterations, and Gauss-Seidel Iterations. For the iteration methods, begin with the guess $(I_1, I_2, I_3) = (0, 0, 0)$. This will give you three additional matrices of size 3 columns by 51 rows for the LU, Jacobi and Gauss-Seidel respectively.

ANSWERS: Should be written out as A2.dat–A4.dat

(c) For the two iteration methods, what is the **average** number of iterations required to solve the given equation with accuracy 10^{-6}. The accuracy constraint should be based upon looking at the norm of the difference between successive iterations. Thus it is required that $\|\vec{x}_{n+1} - \vec{x}_n\| < 10^{-6}$. Save the two answers (for Jacobi first and Gauss-Seidel second) as a row vector with two components.

ANSWER: Should be written out as A5.dat

II Consider the following temperature data taken over a 24-hour (military time) cycle:

75 at 1, 77 at 2, 76 at 3, 73 at 4, 69 at 5, 68 at 6, 63 at 7, 59 at 8, 57 at 9, 55 at 10, 54 at 11, 52 at 12, 50 at 13, 50 at 14, 49 at 15, 49 at 16, 49 at 17, 50 at 18, 54 at 19, 56 at 20, 59 at 21, 63 at 22, 67 at 23, 72 at 24.

(a) Fit the data with the parabolic fit

$$f(x) = Ax^2 + Bx + C \tag{1}$$

and calculate the E_2 error. Use POLYFIT and POLYVAL to get your results. Evaluate the curve $f(x)$ for $x = 1 : 0.01 : 24$ and save this in a column vector.

ANSWER: The error and curve should be written out as A6.dat and A7.dat respectively

(b) Use the INTERP1 and SPLINE command to generate an interpolated approximation to the data for $x = 1 : 0.01 : 24$. Save these two results as column vectors.

ANSWER: Should be written out as A8.dat and A9.dat

(c) Develop a Least-Squares algorithm and calculate E_2 for:

$$y = A\cos Bx + C \tag{2}$$

(Hint: use the MATLAB FMINSEARCH command to help. YOUR INITIAL GUESS IS CRITICAL! So be sure to plot your results to see if they look right!) Evaluate the curve for $x = 1 : 0.01 : 24$ and save this in a column vector.

ANSWER: Error and curve should be written out as A10.dat and A11.dat respectively