AMATH 301
Homework 3: Autumn 2008

DUE: Thursday, November 6 at 3 a.m.

I Download the file **velocity.dat** from the class webpage. This file will be located next to **301hw3.pdf**. This data contains the velocity (meters/second) as a function of time (seconds). Find the acceleration as a function of time. To do this, you will need to differentiate the data as a function of time. Do this in the following ways:

(a) Use an $O(\Delta t^2)$ accurate scheme on the raw data.

ANSWER: Should be written out as a row vector A1.dat

(b) Fit a spline through the data with $t = 0 : 0.01 : 30$ and find the $O(\Delta t^2)$ and $O(\Delta t^4)$ results. (At boundaries use $O(\Delta t^2)$ accurate forward- and backward-difference scheme.)

ANSWER: Should be written out as row vectors A2.dat and A3.dat

(c) Fit the least-squares curve (AGAIN, YOUR INITIAL GUESS IS CRITICAL! So try using the initial guesses for A, B, C and D being $3, \pi/4, 2/3$ and 32 respectively.)

$$f(t) = A\cos(Bt) + Ct + D \tag{1}$$

through the data points and differentiate the resulting best fit with an $O(\Delta t^2)$ accurate scheme using $t = 0 : 0.01 : 30$.

ANSWERS: Curve and derivative are row vectors A4.dat and A5.dat respectively

II Download the file **velocity.dat** from the class webpage. This file will be located next to **301hw3.pdf**. This data contains the velocity (meters/second) as a function of time (seconds). Find the position as a function of time. To do this, you will need to integrate the data as a function of time. Do this in the following ways:

(a) Use a trapezoidal rule (**CUMTRAPZ**) on the data.

ANSWER: Should be written out as column vector A6.dat

(b) Use a spline with $t = 0 : 0.01 : 30$ and a trapezoidal rule to evaluate the integral.

ANSWER: Should be written out as column vector A7.dat

(c) Fit the least-squares curve $f(t) = A\cos(Bt) + Ct + D$ as with the previous homework using the initial guesses for A, B, C and D being $3, \pi/4, 2/3$ and 32 respectively. Integrate with the **QUAD** and **INLINE** command and give the results for the cumulative integral for $t = 0 : 0.1 : 30$.

ANSWER: Should be written out as column vector A8.dat

III Consider the Van der Pol differential equation (use ode45)

$$y'' + \epsilon(y^2 - 1)y' + y = 0$$

which has the nonlinear damping term $\epsilon(y^2 - 1)y'$.

(a) With $\epsilon = 0.1$, solve the equation for $t \in [0 : 0.5 : 30]$ for initial conditions $y(0) = 0.1$ and $y'(0) = -1$. Repeat with $\epsilon = 1$ and $\epsilon = 20$.

ANSWER: Should be written out as two column matrices A9.dat–A11.dat for $\epsilon = 0.1, 1$ and 20 respectively.

(b) With $\epsilon = 1$, $t \in [0, 30]$ (let MATLAB pick step-size) and initial conditions $y(0) = 5$ and $y'(0) = 0$, solve the equation with the four different integration methods: ode45, ode23, ode113, and ode15s. For each method, use the *diff* and *mean* command to calculate the average step-size Δt taken to solve the problem over $t \in [0, 30]$ for a given tolerance. Control the error tolerance, TOL, in the ode solvers with

```
TOL=1e-4;
OPTIONS = odeset('RelTol',TOL,'AbsTol',TOL);
[T,Y] = ODE45('F',TSPAN,Y0,OPTIONS);
```

Use the following tolerance values: $10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}, 10^{-9}, 10^{-10}$. Plot on a log-log scale the average step-size (x-axis) versus the tolerance (y-axis) for the given tolerance values. Calculate the slopes of these lines with the *polyval* command. Note that the local error should be $O(\Delta t^5)$ and $O(\Delta t^3)$ for ode45 and ode23 respectively. What are the local errors for ode113 and ode15s?

ANSWER: Slopes should be written out as a row vector with four components A12.dat whose components are slopes of $ode45, ode23, ode113$ and $ode15s$ respectively.