

## Final Review

The exam is Monday, June 4, 2007 at 8:30 am. The material will be over the whole course, with more emphasis on post-midterm material. No notes, books or calculators allowed. Bring your mind. I will provide you a list of relevant MATLAB function calls if you need them. In the exam you will be asked to both calculate by hand and **ALSO** write MATLAB code. When you write code for the exam, write it as if I will write a .m file and run your code. Along with the midterm review:

- Numerical Differentiation and Integration (page 38-50 in notes)
  - Round off error and how it can effect computations (idea).
  - Taylor series and how they are used to approximate derivatives.
  - Be able to check the **order** of a certain approximation.
  - Optimal step-size to reduce error.
  - Composite Trapezoidal rule (integrating some function).
  - MATLAB **trapz**, **cumtrapz**, **quad** commands.
- Ordinary Differential Equations (pages 50-78 in notes)
  - Initial Value Problems:
    - \* Euler’s Method and how to time-step with it.
    - \* Local Truncation error of Euler’s method.
    - \* How to check the **order** of a time-step method by reducing the step-size and calculating the error.
    - \* Idea of Runge-Kutta methods (Heun’s method).
    - \* Writing a higher order differential equation as a first order **system**.
    - \* Implementation of MATLAB’s ode solver (**ode45**).
    - \* Understand output from MATLAB’s ode solver (**ode45**).
  - Boundary Value Problems:
    - \* Idea of the shooting method.
    - \* Direct solve method.
- Partial Differential Equations (pages 89-105)
  - Solving PDE with spatial discretization:
    - \* Solving a PDE using spatial discretization and time-stepping.
    - \* Know how to represent a *first derivative* operator as a centered second-order matrix operation.
    - \* Know how to represent a *second derivative* operator as a centered second-order matrix operation.
    - \* **spdiags** command in MATLAB.
    - \* How the spatial discretization converts the PDE into a system of ODEs.
    - \* Implementing MATLAB code to solve the PDE using spatial discretization.

- Solving a PDE spectrally:
  - \* `fft`, `ifft` commands, and what they do.
  - \* The relation of a Fourier Transform of a spatial derivative of some function to the Fourier Transform of that function.
  - \* Taking the Fourier Transform of a PDE.
  - \* How taking the Fourier Transform of a PDE converts it to an ODE for the transform of the function you want.
  - \* Time-stepping the transformed equation to obtain a solution.
  - \* Idea of periodic boundary conditions.

## Example Problems

1. Consider the approximation to the second derivative:

$$\frac{d^2 f(x)}{dx^2} \approx \frac{f(x + \Delta x) - 2f(x) + f(x - \Delta x)}{\Delta x^2}$$

Using a Taylor series approximation, obtain the leading order truncation error in this approximation.

2. Consider the ordinary differential equation:

$$\frac{d^2 u}{dt^2} + \frac{du}{dt} + u^3 = 0$$

- (a) Write this second order ODE as a first order system.
- (b) Write a MATLAB code that uses ODE45 to solve for  $u(t)$ . Use a `tspan = [0, 10]`,  $u(0) = 1$ , and  $u'(0) = 0$ . Include your `rhs.m` file.