**AMath 301**
**Hand Calculations Due Date: Monday, April 23, beginning of class**
**Matlab code Due Date: Submissions by early Tuesday morning, April 24**

# Homework 2

1.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 1 \\ 21 \\ 13 \end{bmatrix}$$

(a) Solve the linear system $Ax = b$ using Gaussian Elimination (by hand) for both $b_1$ and $b_2$.

(b) Compute (by hand) the LU factorization of the matrix $A$. Do not use pivoting.

(c) Use this LU factorization to solve the systems $Ax = b$ for each choice of $b$. Do this by first using forward substitution to solve $Ly = b$ and then back substitution to solve $Ux = y$.

**ANSWERS**: Turn in hand calculations on paper, also check your answer using the backslash command and save the two column vector solutions as A1-A2.dat

2.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

Calculate (by hand) the eigenvalues and eigenvectors for the matrix $A$.

**ANSWERS**: Turn in hand calculations on paper, also check your answers using the eig command and save the eigenvalues as A3-A4.dat

3. Consider the circuit depicted in the figure. By using the following two facts:

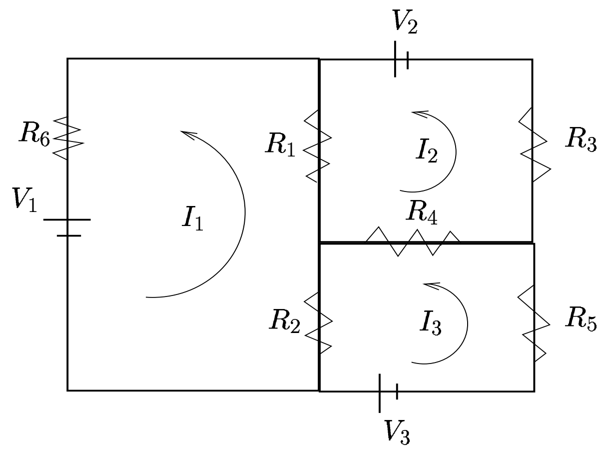
- The voltage drop across a resistor is $V = IR$.
- The sum of all voltage drops in a closed loop sum is zero.

The currents $I_1$, $I_2$, and $I_3$ are determined from the (3 x 3) system:

$$\begin{aligned} R_6 I_1 + R_1(I_1 - I_2) + R_2(I_1 - I_3) &= V_1 \\ R_3 I_2 + R_4(I_2 - I_3) + R_1(I_2 - I_1) &= V_2 \\ R_5 I_3 + R_4(I_3 - I_2) + R_2(I_3 - I_1) &= V_3 \end{aligned}$$

where $R_1 = 20$, $R_2 = 10$, $R_3 = 25$, $R_4 = 10$, $R_5 = 30$, $R_6 = 40$, $V_2 = 0$, $V_3 = 200$ and $V_1$ will be variable. In this form, the associated matrix is strictly diagonal dominant.

(a) Vary $V_1$ from 0 to 100 in steps of 2 (i.e. $V_1 = 0, 2, 4, 6, \ldots, 100$) and calculate $I_1$, $I_2$, and $I_3$ as a function of increasing $V_1$ by solving the system with the backslash command. Save your results in a matrix of 3 columns and 51 rows where the first, second and third cloumn are $I_1$, $I_2$, and $I_3$ respectively.
**ANSWER**: Written out as A5.dat




(b) Repeat part (a), but now solve it with three additional methods: LU decomposition, Jacobi Iterations, and Gauss-Seidel Iterations. For the iteration methods, begin with the guess $(I_1, I_2, I_3) = (0, 0, 0)$. Also for the two iteration methods, the accuracy constraint should be based upon looking at the norm of the difference between successive iterations. Thus it is required that $\|\vec{x}_{n+1} - \vec{x}_n\|_2 < 10^{-6}$. This will give you three additional matrices of size 3 columns and 51 rows for the LU, Jacobi and Gauss-Seidel respectively.
**ANSWER**: Written out as A6-A8.dat

(c) For the two iteration methods, what is the **average** number of iterations required to solve the given equation with accuracy $10^{-6}$. Save the two answers (for Jacobi first, and Gauss-Seidel second) as a row vector with two components.
**ANSWER**: Written out as A9.dat

4. Consider the data in **temperature.dat** on the web page. This is temperature data taken over a 24 hour (military time) period.

(a) Fit the data with the parabolic fit

$$f(T) = AT^2 + BT + C$$

and calculate the $E_2$ error. Use POLYFIT and POLYVAL to get your results. Evaluate the **curve** $f(T)$ for $T = 1 : 0.01 : 24$ and save this in a column vector.
**ANSWERS**: The error and **curve** should be written out as A10.dat, A11.dat respectively.

(b) Use the INTERP1 and SPLINE command to generate an interpolated approximation to the data for $T = 1 : 0.01 : 24$. Save your two results as column vectors.
**ANSWERS**: Written out as A12-A13.dat

(c) Develop a Least-Squares algorithm and calculate $E_2$ for:

$$y = A\cos(BT) + C$$

(Hint: use the MATLAB FMINSEARCH command to help. YOUR INITIAL GUESS IS CRITICAL!) Evaluate the curve for $T = 1 : 0.01 : 24$.
**ANSWERS**: The error and curve should be written out as A14.dat, A15.dat respectively.