AMath 301: Homework 3
Due Date: Early Monday morning, May 7th

1. A object falling vertically through the air is subjected to viscous resistance as well as to the force of gravity. Assume that an object with mass m is dropped from a height S_0 and that the height of the object after t seconds is

$$S(t) = S_0 + \frac{mg}{k}t - \frac{m^2 g}{k^2}\left(1 - e^{-kt/m}\right),$$

where $g = -32.17$ ft/s^2, $S_0 = 300$ ft, $m = .25$ lb. and and the air resistance coefficiant is $k = .5$ lb $\cdot$ s/ft.

   (a) Write a fixed point iteration scheme to solve for the time the object hits the ground. Use a convergence criteria which evaluates the function to see if it is greater than your tolerance, which you should take to be 10^{-6}. Use $t_0 = 1$ as an initial guess.
   (*Hint*: Use the iteration function with no logarithm in it. Understand **why** this iteration function converges.)
   ANSWERS: Save the time when the object hits the ground and the number of iterations required for this method to converge in A1.dat and A2.dat respectively.

   (b) Solve the same problem only use Newton's method. Again use a tolerance of 10^{-6} and initial guess of $t_0 = 1$.
   ANSWERS: Save the time when the object hits the ground and the number of iterations required for this method to converge in A3.dat and A4.dat respectively.

2. Solve the nonlinear system

$$\begin{aligned} \sin(x) + 3\cos(x) - 2 &= 0 \\ \cos(x) - \sin(y) + 0.2 &= 0 \end{aligned}$$

using Newton's method. Evaluate the 2-norm criteria with tolerance 10^{-6}. Start with an initial guess of $[x,\ y] = [1,\ 1]$.
ANSWERS: Save the solution as a column vector in A5.dat. Save the number of iterations required for this method to converge in A6.dat. Save the Jacobian evaluated at the converged solution as A7.dat.

3. Download the file **velocity.dat** from the class website. This data contains the velocity (meters/second) as a function of time (seconds). Find the acceleration as a function of time. To do this, you will need to differentiate the data as a function of time. Do this in the following ways:

   (a) Use an $O(\Delta t^2)$ accurate scheme on the raw data.
   ANSWER: Should be written out as a **row** vector in A8.dat

   (b) Fit a spline through the data with $t = 0 : .01 : 30$ and find the $O(\Delta t^2)$ and $O(\Delta t^4)$ results. (At the boundaries, use an $O(\Delta t^2)$ accurate forward- and backward-difference schemes.
   ANSWERS: Should be written out as **row** vectors in A9.dat ($O(\Delta t^2)$) and A10.dat ($O(\Delta t^4)$).

(c) Fit the least squares curve (use fminsearch, the same E2 error definition as in gafit.m on web, and the initial guess $[A,\ B,\ C,\ D] = [3,\ \pi/4,\ 2/3,\ 32]$).

$$f(t) = A\cos(Bt) + Ct + D$$

through the data points and differentiate the resulting best fit curve with an $O(\Delta t^2)$ accurate scheme using $t = 0 : .01 : 30$.
ANSWERS: Curve and derivative should be written as **row** vectors in A11.dat and A12.dat respectively.

4. From the same data file **velocity.dat** we will find the position as a function of time. To do this, we will need to integrate the data as a function of time. Do this in the following ways:

   (a) Use a trapezoidal rule (**CUMTRAPZ**) on the raw data.
   ANSWER: Should be written out as a **column** vector in A13.dat

   (b) Use a spline with $t = 0 : .01 : 30$ and the trapezoidal rule to evaluate the integral.
   ANSWER: Should be written out as **column** vector in A14.dat.

   (c) Fit the least squares curve $f(t) = A\cos(Bt) + Ct + D$ using the initial guess $[A,\ B,\ C,\ D] = [3,\ \pi/4,\ 2/3,\ 32]$. Integrate with the **QUAD** and **INLINE** command and give the results for the cumulative integral for $t = 0 : .01 : 30$.
   ANSWER: Should be written out as a **column** vector in A15.dat

5. Consider the Van der Pol differential equation

$$y'' + \epsilon(y^2 - 1)y' + y = 0.$$

   (a) (Use ODE45) : With $\epsilon = .1$, solve the equation for $t \in [0 : 0.5 : 30]$ for the initial conditions $y(0) = .1$ and $y'(0) = -1$. Repeat for $\epsilon = 1$ and $\epsilon = 20$.
   ANSWERS: Solution (y vector and y' vector) should be written out as a two-column matrix in A16.dat - A18.dat for $\epsilon = .1, 1, 20$ respectively.

   (b) With $\epsilon = 1$, $t \in [0, 30]$ (let MATLAB pick the step-size) and initial conditions $y(0) = 5$ and $y'(0) = 0$ solve the equation with four integration methods: ode45, ode23, ode113, and ode15s. For each method use the *diff* and *mean* command to calculate the average step-size Δt taken to solve the problem for $t \in [0, 30]$ with a given tolerance. Control the error tolerance TOL, in the ODE solvers with

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TOL = 1e-4; OPTIONS = odeset('RelTol',TOL,'AbsTol',TOL);
[T,Y] = ode45('F', tspan,y0,OPTIONS);
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   Use the following tolerance values: 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}, 10^{-9}, 10^{-10}. Plot on a log-log scale the average step-size (x axis) versus the tolerance (y axis) for the given tolerance values. Calculate the slopes of these lines using the **polyfit** command.
   ANSWER: Slopes should be written out as A19.dat - A22.dat for ode45, ode23, ode113, and ode15s respectively.