**AMath 301**
**Due Date: Early Friday morning, June 1st**

# Homework 5

1. Consider the nonlinear partial differential equation known as the Nonlinear Schrödinger Equation (NLS):

$$i\frac{\partial u}{\partial T} + \frac{\partial^2 u}{\partial x^2} + 2|u|^2 u = 0$$

where $i = \sqrt{-1}$ and $u = u(x, T)$ is a function of both $x$ and $T$. Assume periodic boundary conditions with $L = 20$ ($x \in (-10,\ 10)$) and use $N = 128$ for the number of discrete points. For your time-stepping routine, use ode45 with Tspan $= [0 : 1 : 20]$.

(a) Using the initial condition $U(x) = \text{sech}(x)$, solve the PDE using a finite difference method. Using a waterfall plot, view the absolute value of the solution:

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>> waterfall(x,T,abs(u))
```

This is what is called a *soliton* solution to NLS. As this solution propagates in $T$, it holds it's form!

**ANSWERS**: ode45 solution output should be saved as A1.dat

(b) Using the initial condition $U(x) = 2\text{sech}(x)$, solve the PDE using a finite difference method. Using a waterfall plot, view the absolute value of the solution: The solution here is doing what is called a "soliton dance", where it returns to it's initial state after $T = \pi/2$ units.

**ANSWERS**: ode45 solution output should be saved as A2.dat

(c) Using the initial condition $U(x) = \text{sech}(x)$, solve the PDE spectrally using FFTs. Using a waterfall plot, view the absolute value of the **solution** $u$. Compare with (a).

**ANSWERS**: ode45 solution output should be saved as A3.dat.

(d) Using the initial condition $U(x) = 2\text{sech}(x)$, solve the PDE spectrally using FFTs. Using a waterfall plot, view the absolute value of the **solution** $u$. Compare with (b).

**ANSWERS**: ode45 solution output should be saved as A4.dat.

*Note*: Each data file is worth 5 points.