AMath 301
Due Date: Friday, April 4
Submissions due by 5:00 pm. Write as *one* .m-file that generates all data.

Homework 1

1. The following expressions all result in zero:

$$1,000 - \sum_{i=1}^{10,000} 0.1, \qquad 10,000 - \sum_{i=1}^{100,000} 0.1, \qquad 100,000 - \sum_{i=1}^{1,000,000} 0.1$$

Write a MATLAB algorithm to compute each of the above repeated subtractions and compare the answer to the exact answer of zero (i.e. compute the Absolute Error).

ANSWERS: Errors written out as A1.dat - A3.dat

2. Let the following be defined:

$$A=\begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix}, B=\begin{bmatrix} 2 & 3 \\ -2 & -1 \end{bmatrix}, C=\begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, D=\begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 5 & 2 \end{bmatrix}, x=\begin{bmatrix} 2 \\ -2 \end{bmatrix}, y=\begin{bmatrix} 0 \\ 1 \end{bmatrix}, z=\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix},$$

Calculate the following. If you cannot, state why.

(a)$A+B$, (b) $2x+y$, (c) Ax, (d) $B(x+2y)$, (e) Dy, (f) $Dy-z$, (g) AB, (h) BC, (i) CD

ANSWERS: Written out as A4.dat - A12.dat

3. Consider the logistic equation

$$x_{n+1} = \beta x_n(1 - x_n)$$

which was first developed to model the growth and decay of a population of some species. Iterate the equation for the following values of β with $x_0 = .5$:

$$\beta = 0.8, 1.5, 2.8, 3.2, 3.5, 3.65$$

Iterate the equation for each β value and calculate six column vectors (one for each β value) of length 50 which contains x_1 to x_{50}.

ANSWERS: Written out as A13.dat - A18.dat

4. A object falling vertically through the air is subjected to viscous resistance as well as to the force of gravity. Assume that an object with mass m is dropped from a height S_0 and that the height of the object after t seconds is

$$S(t) = S_0 + \frac{mg}{k}t - \frac{m^2 g}{k^2}\left(1 - e^{-kt/m}\right),$$

where $g = -32.17$ ft/s^2 and k represents the coefficient or air resistance in lb $\cdot$ s/ft. Suppose $S_0 = 300$ ft, $m = .25$ lb.

(a) Write a MATLAB function that uses the bisection method to find the time when the object hits the ground. The function should take as *input* the damping value k, and should *output* the time the object hits the ground. Use as your left endpoint $t = 0$ and your right endpoint $t = 50$. Your tolerance should be set to 10^{-5}.

(b) Using this function, find the time the object hits the ground for different damping values ranging from $k = 0.1$ to $k = 1$ in steps of 0.1 (i.e $k = 0.1, 0.2 \cdots 1$). Save these times in a column vector.

ANSWER: Write out as A19.dat