

AMATH 301
Homework 3: Summer 2007

DUE: Tuesday, July 31st at 4am

I Download the file **velocity.dat** from the class webpage. This data contains the velocity (meters/second) as a function of time (seconds). Find the acceleration as a function of time. To do this, you will need to differentiate the data as a function of time. Do this in the following ways:

(a) Use an $O(\Delta t^2)$ accurate scheme on the raw data.

ANSWER: Should be written out as a row vector A1.dat

(b) Fit a spline through the data with $t = 0 : 0.01 : 30$ and find the $O(\Delta t^2)$ and $O(\Delta t^4)$ results. (At boundaries use $O(\Delta t^2)$ accurate forward- and backward-difference scheme.)

ANSWER: Should be written out as row vectors A2.dat and A3.dat

(c) Fit the least-squares curve (AGAIN, YOUR INITIAL GUESS IS CRITICAL!)

$$f(t) = A \cos(Bt) + Ct + D \tag{1}$$

through the data points and differentiate the resulting best fit with an $O(\Delta t^2)$ accurate scheme using $t = 0 : 0.01 : 30$.

ANSWERS: Curve and derivative are row vectors A4.dat and A5.dat respectively

II Again using the data **velocity.dat**, find the position as a function of time. To do this, you will need to integrate the data as a function of time. Do this in the following ways:

(a) Use a trapezoidal rule (**CUMTRAPZ**) on the data.

ANSWER: Should be written out as column vector A6.dat

(b) Use a spline with $t = 0 : 0.01 : 30$ and a trapezoidal rule to evaluate the integral.

ANSWER: Should be written out as column vector A7.dat

(c) Fit the least-squares curve $f(t) = A \cos(Bt) + Ct + D$ as with the previous question. Integrate with the **QUAD** and **INLINE** command and give the results for the cumulative integral for $t = 0 : 0.1 : 30$.

ANSWER: Should be written out as column vector A8.dat

III Consider the Van der Pol differential equation (use **ode45**)

$$y'' + \epsilon(y^2 - 1)y' + y = 0$$

which has the nonlinear damping term $\epsilon(y^2 - 1)y'$.

(a) With $\epsilon = 0.1$, solve the equation for $t \in [0 : 0.5 : 30]$ for initial conditions $y(0) = 0.1$ and $y'(0) = -1$. Repeat with $\epsilon = 1$ and $\epsilon = 20$.

ANSWER: Should be written out as two column matrices A9.dat–A11.dat for $\epsilon = 0.1, 1$ and 20 respectively.

(b) With $\epsilon = 1$, $t \in [0, 30]$ (let MATLAB pick step-size) and initial conditions $y(0) = 5$ and $y'(0) = 0$, solve the equation with the four different integration methods: **ode45**, **ode23**, **ode113**, and **ode15s**. For each method, use the *diff* and *mean* command to calculate the average step-size Δt taken to solve the problem over $t \in [0, 30]$ for a given tolerance. Control the error tolerance, **TOL**, in the ode solvers with

```
TOL=1e-4;
OPTIONS = odeset('RelTol',TOL,'AbsTol',TOL);
[T,Y] = ODE45('F',TSPAN,Y0,OPTIONS);
```

Use the following tolerance values: 10^{-4} , 10^{-5} , 10^{-6} , 10^{-7} , 10^{-8} , 10^{-9} , 10^{-10} . Plot on a log-log scale the average step-size (x-axis) versus the tolerance (y-axis) for the given tolerance values. Calculate the slopes of these lines with the *polyval* command. Note that the local error should be $O(\Delta t^5)$ and $O(\Delta t^3)$ for ode45 and ode23 respectively. What are the local errors for ode113 and ode15s?

ANSWER: Slopes should be written out as a row vector with four components A12.dat whose components are slopes of *ode45*, *ode23*, *ode113* and *ode15s* respectively.

IV Consider the Lorenz differential equation (use ode45):

$$\frac{dx}{dt} = \sigma(-x + y) + \gamma \cos(t) \quad \frac{dy}{dt} = rx - y - xz \quad \frac{dz}{dt} = -bz + xy \quad (2)$$

with $b = 8/3$, $\sigma = 10$, $t \in [0 : 1 : 100]$ and $x(0) = y(0) = z(0) = 1$.

(a) Solve the system with $\gamma = 0$ and for $r = 0.5, 2, 9, 15, 22, 25, 28$. Compare the dynamics for each of the r values and plot the results using PLOT3 and GRID ON.

ANSWER: Should be written out as three column matrices A13.dat–A19.dat for $r = 0.5, 2, 9, 15, 22, 25$ and 28 respectively.

(b) Repeat (a) with $\gamma = 1$

ANSWER: Should be written out as three column matrices A20.dat–A26.dat for $r = 0.5, 2, 9, 15, 22, 25$ and 28 respectively.