AMATH 301
Homework 4: Summer 2007

DUE: Wednesday, August 15 at 4am.

I Consider the boundary value problem:

$$\frac{d^3y}{dx^3} + \alpha\frac{dy}{dx} + \sin(y) = \cos(x)\sin(x) \tag{1}$$

with boundary values $y(0) = 1$ and $y(10) = 1$ and initial conditions $y'(0) = 0$ and $y''(0) = 0$.

Solve the boundary value problem using the shooting method. Use `ode45` and let MATLAB choose step size. Use 1e-6 as the tolerance. For the initial guesses, use high=2 and low=1.5.

ANSWER: Save the values of $y(x)$ as a column vector in A1.dat. Save the value of α which solves the problem in A2.dat.

II Consider the forced harmonic oscillator:

$$\frac{d^2y}{dt^2} = 2t\sin(3t)\frac{dy}{dt} + e^{\cos(t)}y + u(t) \tag{2}$$

where the forcing function is a Heaviside-type function which "turns on" at time $t = 5$:

$$u(t) = \begin{cases} 0 & \text{if } 0 \leq t < 5, \\ 8\cos(t) & \text{if } t \geq 5. \end{cases} \tag{3}$$

The problem also has boundary values $y(0) = 3$ and $y(10) = -2$.

Solve the boundary value problem using the direct solve method. Use a discretization in time with step size $dt = 0.1$.

ANSWER: The direct solve method involves setting up a linear system in the form $Ax = b$. Save the matrix A in A3.dat. Save the column vector b in A4.dat. Save the values of the final solution $y(t)$ as a column vector in A5.dat.

III Download the data file noisydata.dat from the class website. This file contains a noisy signal with two major wavenumber components (the wavenumber of the function $y = \sin(nx)$ is n).

Use the Fast Fourier Transform to pinpoint these two wavenumbers (note that n and $-n$ represent the same wavenumber, use the positive number for the answer). Assume that the data was taken from time $t = -\pi$ to $t = \pi$.

ANSWER Save the two wavenumbers as a row vector with two components (in numerical order) in A6.dat.

IV Consider the reaction-diffusion system

$$U_t = (1 - U^2 - V^2)U + \beta(U^2 + V^2)V + D_1\nabla^2 U$$
$$V_t = -\beta(U^2 + V^2)U + (1 - U^2 - V^2)V + D_2\nabla^2 V$$

where $\beta = 1$, $D_1 = D_2 = 0.1$, $\nabla^2 = \partial_x^2 + \partial_y^2$ and $x, y, \in [-10, 10]$. Assume periodic boundary conditions and use $n = 64$ for the number of discrete points.

Initial Conditions Start with spiral initial conditions in U and V.

```
[X,Y]=meshgrid(x,y);  m=1; % number of spirals
u=tanh(sqrt(X.^2+Y.^2)).*cos(m*angle(X+i*Y)-(sqrt(X.^2+Y.^2)));
v=tanh(sqrt(X.^2+Y.^2)).*sin(m*angle(X+i*Y)-(sqrt(X.^2+Y.^2)));
```

Discretize the PDE and use **ode45** to advance the solution in time for $tspan = 0 : 0.5 : 10$.

(a) Solve the system using a second order accurate finite-difference scheme.

ANSWER: Save the ode45 solution output twice, in A7.dat and A8.dat.

(b) Repeat with the number of initial spirals equal to 3 ($m = 3$).

ANSWER: Save the ode45 solution output twice, in A9.dat and A10.dat.