AMath 301

Final Review

The exam is Thursday, March 20, 2008 at 8:30 am. The material will be over the whole course, with more emphasis on post-midterm material. No notes, books or calculators allowed. Bring your mind. I will provide you a list of relevant MATLAB function calls if you need them. In the exam you will be asked to both calculate by hand and **ALSO** write MATLAB code. When you write code for the exam, write it as if I will write a .m file and run your code.

Along with the midterm review:

- Ordinary Differential Equations (pages 50-78 in notes)
  - Initial Value Problems:
    - Euler's Method and how to time-step with it.
    - Local Truncation error of Euler's method.
    - How to check the **order** of a time-step method by reducing the step-size and calculating the error.
    - Idea of Runge-Kutta methods (Heun's method).
    - Writing a higher order differential equation as a first order **system**.
    - Implementation of MATLAB's ode solver (**ode45**).
    - Understand output from MATLAB's ode solver (**ode45**).
  - Boundary Value Problems:
    - Idea of the shooting method for a linear second order problem (see handout).
    - Idea and implementation of shooting method for nonlinear problems.
    - Direct solve method.
- Partial Differential Equations (pages 89-105)
  - Solving PDE with spatial discretization:
    - Solving a PDE using spatial discretization and time-stepping.
    - Know how to represent a *first derivative* operator as a centered second-order matrix operation.
    - Know how to represent a *second derivative* operator as a centered second-order matrix operation.
    - **spdiags** command in MATLAB.
    - How the spatial discretization converts the PDE into a system of ODEs.
    - Implementing MATLAB code to solve the PDE using spatial discretization.
  - Solving a PDE spectrally:
    - **fft**, **ifft** commands, and what they do.
    - The relation of a Fourier Transform of a spatial derivative of some function to the Fouier Transform of that function.
    - Taking the Fourier Transform of a PDE.
    - How taking the Fourier Transform of a PDE converts it to an ODE for the transform of the function you want.

* Time-stepping the transformed equation to obtain a solution.
* Idea of periodic boundary conditions.

Example Problems

1. Consider the approximation to the second derivative:

$$\frac{d^2 f(x)}{dx^2} \approx \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2}$$

   Using a Taylor series approximation, obtain the leading order truncation error in this approximation.

2. Consider the ordinary differential equation:

$$\frac{d^2 u}{dt^2} + \frac{du}{dt} + u^3 = 0$$

   (a) Write this second order ODE as a first order system.

   (b) Write a MATLAB code that uses ODE45 to solve for $u(t)$. Use a tspan $= [0, 10]$, $u(0) = 1$, and $u'(0) = 0$. Include your rhs.m file.