

# 1 General Method for solving Ordinary Differential Equations

- Classify the equation (Is it something we've studied?)
- Put the equation in standard form, depending on the classification.
- Apply appropriate method to get a general solution.
- Apply initial conditions to get a specific solution.

Notes: Efficient application of the various methods comes with practice. Sometimes, equations can be classified more than one way. Choose the approach with which you are most comfortable.

## 2 Equations We've studied

- First Order
  - Linear
  - Nonlinear
  - Separable
  - Exact
  - Homogenous
  - Bernoulli
- Second Order
  - Linear Constant Coefficient Homogenous
  - Linear Constant Coefficient Inhomogenous

## 3 Review Problems

1.  $y + xy' = 0$
2.  $(x^2 - 1)y' + x = 0$ ,  $y(2) = 1$
3.  $9y'' + y - (xe^x)^{1/3} = 6y'$
4.  $y' - y - y^2 = 0$ ,  $y(0) = 1$
5. Show that  $y + (2x - ye^y)y' = 0$  is not exact. Then show that when multiplied by the integrating factor  $\mu(x, y) = y$ , the equation becomes exact. Solve.
6. For what values of  $x$  and  $y$  does there exist a unique solution to  $y' = \sqrt{y^3 - t + 2}$ ?
7.  $9\frac{d^2x}{dt^2} + x = 0$
8.  $y'' + 2y' + 5y = 5$
9.  $\frac{d^2x}{dt^2} - x = \frac{1}{t}$

10. Linear homogeneous equations of  $n$ th order are solved just like 2nd order equations. Try to solve  $y'''' - y = 0$ . Your solution should have 4 constants, since this is a fourth order equation.
11.  $2y^2 + 2t + 1 = -(4y^3 + 4xy)y'$
12. Classify the following equations but don't solve:
  - Airy's Equation:  $\frac{d^2y}{dx^2} = xy$
  - Duffing's Equation:  $y'' + ay + y^3 = b$
  - Wave Equation:  $\frac{\partial^2 y}{\partial x^2} - \frac{\partial^2 y}{\partial t^2} = 0$
  - $xy' = y \sin(\cos(y/x))$
13.  $y'' + 8y' + 16y = 2e^{-4x}$ ,  $y(0) = 1$ ,  $y'(0) = 1$
14.  $y' = \frac{-3x^2y^2 - 1}{2x^3y - 5y^4}$ ,  $y(1) = 2$
15.  $xy' + x^2y = \sin(x)$
16.  $y'' - 4y' + 53 = 0$ ,  $y(\pi/7) = 0$ ,  $y'(\pi/7) = 1$
17.  $y'' - 14y' + 53 = e^{-x}$
18.  $x^2y'' - 3xy' + 3y = \ln(x)$  is a linear equation. Show that  $x^3$  and  $x$  are both solutions to the homogeneous equation. Find a general solution for the inhomogeneous equation.
19.  $x^2y'' + xy' - 4y = 3 + x^2$  is also linear. Show that  $x^{-2}$  and  $x^2$  are solutions to the homogeneous equation. Find a general solution for the inhomogeneous equation.
20. Use the reduction of order method discussed in class to find a second solution to the linear equation  $x^2y'' - 2xy' + 2y = 4x^2$ , given that  $y(x) = x$  is one solution.