Exam 2 Topics

- Variation of Parameters and Greens Functions
- Laplace transforms
- Series solutions

Review Problems

1. Solve $y'' + 4y' + 4y = xe^{-2x}$, $y(0) = 0$, $y'(0) = 1$ with Laplace transforms.
2. Solve $y'' + 4y' + 4y = xe^{-2x}$, $y(0) = 1$, $y'(0) = 0$ with Variation-of-parameters.
3. Solve $y'' + 8y' + 41y = f(x)$

$$f(x) = \begin{cases} 0, & x \leq 1 \\ x-1, & x \leq 3 \\ 2, & x > 3 \end{cases}$$

4. Use the ratio test to check the following series for convergence.
   - $\sum_{k=0}^{\infty} \frac{(t-2)^k}{(3k+1)^2}$
   - $\int_0^{\infty} \frac{e^{-t}}{1+xt} dt = \sum_{n=0}^{\infty} (-1)^n n! x^n$
5. Discuss approaches to the solution of the following problem:

$$y'' - xy = sin(x)$$

6. Classify any singular points of the Bessel Equation

$$x^2y'' + xy' + (x^2 - \nu^2)y = 0$$

7. Solve $y'' + 6y' + 58y = \frac{\sin x}{x}$.
8. A circuit starts without any charge or current, before it is turned on. The circuit has an inductance of 3 Henries, a capacitance of 3 Farads, and a resistance of 1 Ohm. 27 volts is applied to the circuit. What is the charge on the capacitor 5 seconds and 10 seconds after the voltage is applied?
9. Relationships
   - Solve $y'' - 36y = \delta(x)$, $y(0) = 0$, $y'(0) = 0$ with Laplace transforms. Call this solution $w_a(x)$.
   - Solve $y'' - 36y = e^{-x}$, $y(0) = 0$, $y'(0) = 0$ with undetermined coefficients. Call this solution $w_b(x)$.
   - Show $w_b(x) = e^{-x} * w_a(x)$.