Final Exam Topics(emphasis will be place on newer material)

- Homogenious Linear Systems of 1st Order equations
  - Matrix algebra, determinant, trace, eigenvalues, eigenvectors
  - General solutions by eigenvalues and eigenvectors
  - Initial value problems
  - Phase-plane diagrams: identification of saddle's, node's, and spiral's
- Autonomous Non-Linear Systems
  - Identification of Steady-States
  - Orbit and Nullcline construction
  - Linearization at Steady-States
- First-Order Equations
  - Linear equations, and Bernoulli equations that can be turned into linear equations
  - Separable equations, and Homogeneous equations that can be made separable
  - Exact equations
- Second-Order Constant Coefficient Linear Equations
  - Homogeneous equations
  - Inhomogeneous equations and Undetermined Coefficients
- Identification of any of the general classes of equations at which we've looked.

Review Problems

1. Consider the system

$$\dot{x_1} = b^2 x_2$$
$$\dot{x_2} = -a^2 x_1$$

   - Show that the orbits of this system are ellipses if a and b are real numbers, using the method in Section 9.2 and discussed in class.
   - Solve the equations with initial conditions $x_1 = 0,\ x_2 = 1$
   - Check that your solution satisfies the orbit equation.

2. Solve and draw a phase-plane for $\frac{d\vec{x}}{dt} = \begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix} \vec{x}$

3. Prove that the vector $< 1,2,3 >$ is an eigenvector of $M = \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{bmatrix}$.

   What is the corresponding eigenvalue? Also show that $\det(M^2) = \det(M)^2$.

4. Consider $\dot{x} = 4 - 2y,\ \dot{y} = 12 - 3x^2$.
   - Draw the nullclines and steady-states.
   - Linearize around $(x, y) = (2, 2)$ and show that it is a saddle point.

5. Classify each of the following problems, and discuss how you would approach solving it.
   - $\frac{d^2g}{dt^2} + 4\frac{dg}{dt} + 4g = 0,\ g(-1) = 3,\ \frac{dg}{dt}(-1) = 0$
   - $\frac{d^2g}{dt^2} + 4\frac{dg}{dt} + 4 = 0$
   - $y' + x^2 y = e^x$
   - $y'' + y' + x^2 y = 0$
   - $y' = \frac{-2x - y^2}{2xy}$
   - $\dot{x} = \frac{t^2 - 3x^2}{2xt}$
   - $t^2\frac{dy}{dt} + 2y - y^2 = 0$
   - $t^2\frac{dy}{dt} + 2ty - y^2 = 0,\ y(1) = 1$
   - $\dot{x} = 5x,\ \dot{y} = 2x$
   - $\dot{S} = S(1 - S),\ \dot{R} = SR$
   - $\ddot{z} + z - \varepsilon z^3 = 0$
   - $\dot{y}^2 - 2\dot{y} + 4y = 4x$

6. Solve any of the above problems you can.