One-Sided Green's Functions for Common 2nd Order Linear Ordinary Differential Equations

Consider the inhomogenous equation

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = f(x)$$

with homogeneous solution $y(x) = C_1 y_1(x) + C_2 y_2(x)$ Then there exists a particular solution $y_p(x)$ of the form

$$y_p(x) = \int_0^x \frac{y_1(u)y_2(x) - y_1(x)y_2(u)}{y_1(u)y_2'(u) - y_1'(u)y_2(u)} f(u)du = \int_0^x g(x,u) f(u)du,$$

where the Green's function $g(x,u)$ is known from the following table.

Characteristic Equation	Green's Function $g(x,u)$
$\lambda - a$	$e^{a(x-u)}$
λ^2	$x - u$
$(\lambda - a)^2$	$(x-u)e^{a(x-u)}$
$(\lambda - a)(\lambda - b)$	$\frac{e^{a(x-u)} - e^{b(x-u)}}{a-b}$
$\lambda^2 + b^2$	$\frac{1}{b}\sin[b(x-u)]$
$\lambda^2 - b^2$	$\frac{1}{b}\sinh[b(x-u)]$
$\lambda^2 - 2a\lambda + a^2 + b^2$	$\frac{1}{b}e^{a(x-u)}\sin[b(x-u)]$
$\lambda^2 - 2a\lambda + a^2 - b^2$	$\frac{1}{b}e^{a(x-u)}\sinh[b(x-u)]$
$x^2\lambda^2 + x\lambda - b^2$	$\frac{u}{2b}\left(\frac{x^b}{u^b} - \frac{u^b}{x^b}\right)$