AMATH 351 Homework 1

Due January 14, 2009 (W)

Section1.1 15,16,17,18,19,20

Section1.3 1,2,3,4,5,6,13,14,17,19

Section2.2 6,18,32

*For figures and Problem 30, download them on the course webpage

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Section 1.1

Consider the following list of differential equations, some of which produced the direction fields shown in Figure 1.1.5 through 1.1.10. In each of Problems 15 through 20 identify the differential equation that corresponds to the given direction field.

(a) $y' = 2y - 1$
(b) $y' = 2 + y$
(c) $y' = y - 2$
(d) $y' = y(y + 3)$
(e) $y' = y(y - 3)$
(f) $y' = 1 + 2y$
(g) $y' = -2 - y$
(h) $y' = y(3 - y)$
(i) $y' = 1 - 2y$
(j) $y' = 2 - y$

15. The direction field of Figure 1.1.5.
16. The direction field of Figure 1.1.5.
17. The direction field of Figure 1.1.5.
18. The direction field of Figure 1.1.5.
19. The direction field of Figure 1.1.5.
20. The direction field of Figure 1.1.5.

Section 1.3

In each of Problems 1 through 6 determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

1. $t^2 \frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin t$
2. $(1 + y^2)\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t$
3. $\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 1$
4. $\frac{dy}{dt} + ty^2 = 0$
5. $\frac{d^2y}{dt^2} + \sin(t + y) = \sin t$
6. $\frac{d^3y}{dt^3} + t\frac{dy}{dt} + (\cos^2 t)y = t^3$

In each of the following problem verify that each given function is a solution of the differential equation.

13. $y'' + y' = \sec t, \quad 0 < t < \pi/2; \qquad y = (\cos t) \ln \cos t + t \sin t$

14. $y' - 2ty = 1; \qquad y = e^{t^2} \int_0^t e^{-s^2} ds + e^{t^2}$

In the following problem determing the value of r for which the given differential equation has solutions of the form $y = e^{rt}$.

17. $y'' + y' - 6y = 0$

In the following problem determine the value of r for which the given differential equation has solutions of the form $y = t^r \quad for \quad t > 0$.

19. $t^2y'' + 4ty' + 2y = 0$

Section 2.2

Solve the given differential equation

6. $xy' = (1 - y^2)^{1/2}$

In the following problem,

(a) Find the solutionof the given initial value problem in expilicit form.

(b) Plot the graph of the solution.

(c) Determine (at least approximately) the interval in which the solution is defined.

18. $y' = (e^{-x} - e^x)/(3 + 4y), \quad y(0) = 1$

The method outlined in Problem 30 can be used for any homogeneous equation. That is, the substitution $y = xv(x)$ transforms a homogeneous equation into a separable equation. The latter equation can be solved by direct integration, and then replacing v by y/x gives the solution to the original equation. In Problem 32:

(a) Show that the given equation is homogeneous.

(b) Solve the differential equation.

(c) Draw a direction field and some integral curves. Are they symmetric with respect to the origin?

32. $\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$