AMATH 351 Homework 2

Due January 21, 2009

Section2.1 14,16,19,31,40

Section2.4 3,12,15,29

Section2.6 8,21,23,24

Section 2.1

In each of Problems 13 through 20 find the solution of the given initial value problem.

14. $y' + 2y = te^{-2t}, \qquad y(1) = 0$
16. $y' + (2/t)y = (\cos t)/t^2, \qquad y(\pi) = 0, \quad t > 0$
19. $t^3 y' + 4t^2 y = e^{-t}, \qquad y(-1) = 0, \quad t < 0$
31. Consider the initial value problem

$$y' - \frac{3}{2}y = 3t + 2e^t, \qquad y(0) = y_0.$$

Find the value of y_0 that separates solutions that grow positively as $t \to \infty$ from those that grow negatively. How does the solution that corresponds to this critical value of y_0 behave as $t \to \infty$?

In Problem 40 use the method of Problem 38 to solve the given differential equation.

40. $y' + (1/t)y = 3\cos 2t, \qquad t > 0$

Section 2.4

In each of Problems 1 through 6 determine (without solving the problem) and interval in which the solution of the given initial value problem is certain to exist.

3. $y' + (\tan t)y = \sin t, \qquad y(\pi) = 0$

In each of Problems 7 through 12 state where in the ty-plane the hypotheses of Teorem 2.4.2 are satisfied.

12. $\frac{dy}{dt} = \frac{(\cot t)y}{1+y}$

In each of Problems 13 through 16 solve the given initial value problem and determine how the interval in which the solution exists depends on the initial value y_0.

15. $y' + y^3 = 0, \qquad y(0) = y_0$

Bernoulli Equations. Sometimes it is possible to solve a nonlinear equation by making a change of the dependent variable that converts it into a linear equation. The most important such equation has the form

$$y' + p(t)y = q(t)y^n.$$

and is called a Bernoulli equation after Jakob Bernoulli. Problem 27 through 31 deal with equations of this type.

29. $y' = ry - ky^2$, $r > 0$ and $k > 0$. This equation is important in population dynamics and is discussed in detail in Section 2.5.

Section 2.6

Determin whether each of the equations in Problems 1 through 12 is exact. If it is exact, find the solution.

8. $(e^x \sin y + 3y)dx - (3x - e^x \sin y)dy = 0$

Show that the equations in Problems 19 through 22 are not exact but become exact when multiplied by the given integrating factor. Then solve the equations.

21. $ydx + (2x - ye^y)dy = 0, \qquad \mu(x, y) = y$

23. Show that if $(N_x - M_y)/M = Q$, where Q is a function of y only, then the differential equation

$$M + Ny' = 0$$

has an integrating factor of the form

$$\mu(y) = \exp \int Q(y)dy.$$

24. Show that if $(N_x - M_y)/(xM - yN) = R$, where R depends on the quantity xy only, then the differential equation

$$M + Ny' = 0$$

has an integrating factor of the form $\mu(xy)$. Find a general formula for this integrating factor.