Devastating Roundoff Error

Tim Chartier, Davidson College

Using the electrical states of on and off, computers count in base 2. Since a computer has limited memory, not all numbers can be represented exactly. As such, some numbers must be rounded to fit into memory. Consequently, many arithmetic operations on a computer are not performed exactly. When some calculations are performed millions of times, the results of small errors can accumulate into large errors.

In the 1991 Gulf War, the Patriot missile defense system failed due to roundoff error. The troubles stemmed from a computer that performed the tracking calculations with an internal clock whose integral values were converted to decimal by multiplying the binary approximation to 0.1 which was:

$$0.1_{10} = 0.00011001100110011001100_2 = \frac{209715}{2097152}.$$

Note, the error in this calculation is only 0.0001%, a seemingly insignificant amount, since

$$\frac{1}{10} - \frac{209715}{2097152} = \frac{1}{10485760}.$$

However, the timing register incremented at intervals of 0.1 seconds. Therefore, the accumulated errors were proportional to the time on the clock since after x hours the accumulated error was:

$$\left(\frac{1}{10} - \frac{209715}{2097152}\right)(3600 \cdot 10 \cdot x) = \frac{225}{65536}x.$$

Therefore, after eight hours, the system would accumulate an error of 0.0275 seconds. During the war, a Scud missile launched from Iraq and at the same time the internal clock in the defense system had run for 100 hours accumulating an error of 0.3433 seconds, which was the amount of time such a missile could travel more than half a kilometer.

This discrepancy resulted in the Scud missile slipping through the defense system and detonating on barracks, killing 28 people.

FURTHER READING: Eric W. Weisstein. "Roundoff Error." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/RoundoffError.html, and R. Skeel, "Roundoff Error and the Patriot Missile." SIAM News 25, 11, Jul. 1992.