AMATH 383

Introduction to Continuous Modeling

http://www.amath.washington.edu/courses/383-autumn-2003

Autumn 2003 (SLN 1186)
Days: M, W, F
Time: 11:30-12:20
Location: MEB 103

Instructor

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Course Description and Policies

Course Description: Introductory survey of applied mathematics with emphasis on modeling of physical and biological problems in terms of differential equations. Formulation, solution, and interpretation of the results.

Text: The course will follow lecture notes that I have prepared and continue to adapt for this course. They will be posted on the course web-site. Class lectures will parallel and supplement this material, but students should read and study the lecture notes prior to discussion in class.

Supplemental References:
Several textbooks discuss many of the topics in the class.

- *Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic Flow*, by R. Haberman, republished by SIAM in 1998.
- *Differential Equations and Their Applications*, by M. Braun, published by Springer-Verlag (I think its up to 5 editions now).
- *Nonlinear Dynamics and Chaos*, by S. H. Strogatz, published by Addison Wesley in 1995.

Also, there are lecture notes from previous courses by Professor K. K. Tung that also cover related topics that you can find on the internet.

Responsibilities:
Learning is the responsibility of the student. My task as an instructor is as a facilitator for that learning. I will provide lecture notes for students to read, prepare and lead class discussions designed to explain, demonstrate and expand on concepts from the lecture notes, prepare assignments that will encourage learning and comprehension to take place, and provide additional outside assistance through office hours and internet assistance. Your task as the student is to engage in the course to learn and master the course material. You will read the lecture notes critically, attend class as an active participant, seek direction and assistance in the learning process, and accomplish the required work.

Tasks and Grading:
In order to assess student performance and mastery of course topics, there will be four specific tasks throughout the term: (1) Homework, (2) In-class quizzes, (3) Term project proposal, and (4) Term project. The weights of these tasks on the final grade will be 50% for homework, 20% for quizzes, 5% for the term project proposal, and 25% for the term project. Specifics requirements for each of these are listed below.

Effective Grade	4.0	3.7	3.4	3.0	2.7	2.4	2.0	1.7	1.4	1.0	0.7
Homework Score	90%	85%	80%	75%	70%	65%	60%	55%	50%	45%	40%
Quiz Objectives	100%	95%	90%	85%	80%	75%	70%	65%	60%	55%	50%

Homework: The purpose of homework is two-fold: to practice using concepts from the course and to demonstrate mastery. Consequently, homework will include warm-up exercises as well as required assignment problems. Only the required problems will contribute toward the grade, as warm-ups may actually involve recreating solutions based on the lectures and notes. Solutions will require demonstrating mathematical accuracy as well as ability to communicate and relate the mathematical concepts to the modeled physical problem.

Homework must be neat, with the student's name clearly **printed**, and pages securely attached with a staple or folded in half. Be sure that the problems are clearly labeled. Homework should represent your final work, so do not include preliminary calculations that should have been on scratch paper. The concepts of the course rely on a steady progression of understanding, so it is important to stay caught up in the course. In addition, it is important to demonstrate the ability to complete assignments in a timely manner. Homework will typically be due on Wednesdays, by 4:00 pm, to be turned in at Jan Medlock's mailbox in Guggenheim 408. Homework will not be accepted later than two days following the due date, with a 10% deduction for each day late before 4:00 pm.

Grading will focus on accuracy as well as communication. Because this is a modeling course, you should make sure that you relate mathematical results back to the original system under study. For problems that ask for interpretation, use complete sentences and proper language. Recognizing that homework is practice and practice often falls short of perfection, the contribution towards the final grade.

Quizzes: I am exploring the use of objective-based learning with the challenge that course assessment is based on a grade. Instead of having extensive examinations, we will use regular in-class quizzes to provide opportunities to demonstrate mastery of the course learning objectives. These quizzes will be short exercises based on the topics listed in the course syllabus. A final quiz grade will be based on the number of objectives for which the student demonstrates mastery. Students may demonstrate mastery for missed concepts either through subsequent quizzes or through discussion with the instructor.

Term Projects: See the separate handout for an in-depth description of the the term project. Each student needs to select a project, and prepare a type-written proposal, due no later than April 25. The final written project is due the last day of lectures, December 10. Work on the project should be ongoing throughout the term, and not saved for the last week! Groups may work together on the development and solution of projects, but each individual must write her/his own project report.

Optional Writing Credit: See the term project handout for details. The required draft reports for writing credit will be due November 26 to allow adequate time for corrections. This should represent a completed, or nearly completed project, as the goal of the draft is primarily to improve writing.

Collaboration:
Collaboration and student interaction on homework and the term project is encouraged as a tool for cooperative learning. However, individual mastery and personal understanding of the course material is the primary objective of the course. Consequently, students must personally compose their own solutions and explanations, demonstrating their understanding of the mathematical concepts and their physical interpretation.

Syllabus of Planned Topics

The following lists summarize the techniques that students should learn, in addition to a list of example situations where such techniques will be demonstrated. Starred items will depend on the time available and the priorities for the class as the term progresses.

Techniques of Analysis

1. Intuition about Differential Equations
   - (a) Characterization of curve or trajectory
   - (b) Rate of change/slope of curve
   - (c) Role of initial conditions
   - (d) Differentials and Euler's numerical method
   - (e) Properties of graphs
     - i. Extreme values as turning points
     - ii. Concavity and acceleration
     - iii. Points of inflection – switch from accelerate to decelerate
     - iv. Asymptotes
       - A. Vertical – singularity (explosion to infinity in finite time)
       - B. Horizontal – leaving or approaching an equilibrium
       - C. Other – characterize asymptotic behavior
   - (f) Purpose of modeling
     - i. What is the question?
       - A. Prediction – quantitative accuracy
       - B. Exploration – qualitative accuracy
     - ii. Model simplification

iii. Parameter determination

2. Dimensional Analysis
   (a) Dimensions and Units
   (b) Dimensionally homogeneous equation
   (c) Dimensionless vs Nondimensional
   (d) Natural scales
   (e) How to nondimensionalize equations
   (f) Free parameters

3. Graphical Analysis of Differential Equations
   (a) Slope field (1-d)
   (b) Growth rate function
      i. Phase line, direction of growth, and sign of growth rate
      ii. Interpretation of zeros (ie roots) as equilibria
      iii. Determining stability of equilibria (usually by slope)
      iv. Relative extrema in growth rate as points of inflection of solution curves
      v. Relation to slope field
   (c) Direction field (2-d)
   (d) Phase-plane diagrams (2-d)
   (e) Isoclines
   (f) Integral curves
   (g) Orbits

4. Numerical Analysis of Differential Equations
   (a) Generating vector fields (e.g. slope field, direction field)
   (b) Plotting solutions as graphs and as projections
   (c) Plotting solutions in phase plane
   (d) Awareness of different tools
      i. Java tools
      ii. High-level mathematical tools
         A. Mathematica, Maple
         B. Matlab
         C. octave, scilab
      iii. Low-level mathematical tools
         A. mathematical libraries (e.g. GSL)

5. Analysis of Equilibria
   (a) Nullclines
   (b) Stability

(c) Linear systems
    i. Eigenvalues and Eigenvectors (2-d only)
    ii. General solutions
(d) Classification of Equilibria
    i. Node (stable and unstable)
    ii. Focus (stable and unstable)
    iii. Center
    iv. Saddle point
(e) Bifurcation Diagrams
(f) Bifurcation Classification (1 free parameter)
    i. transcritical bifurcation
    ii. saddle-node bifurcation (aka fold, tangent)
    iii. pitch-fork bifurcation
    iv. Hopf bifurcation*

6. Conservative Systems
    (a) Relation to integral curves
    (b) Energy: Exchange between Potential and Kinetic Energy
    (c) Use of potential energy function and phase plane diagrams
    (d) Impact of damping (Lyapunov functions)
    (e) Impact of forcing (potential for resonance)

7. Reducing Complexity: ODEs from PDEs
    (a) Function series (e.g. Fourier series) expansion*
    (b) Method of characteristics*

Formulating Differential Equation Models and Examples

1. Mass Balance
    (a) Rate = Flow In - Flow Out
    (b) Examples
        i. Changing volumes, e.g. evaporation, water flow
        ii. Concentration changes, e.g. diluting salt water

2. Population Growth
    (a) Per capita rates: birth and death
    (b) Harvesting (external increase to death rate)
    (c) Resource dependence
    (d) Examples

        i. Constant per capita rates – exponential growth/decay
        ii. Density or resource dependent rates – logistic growth and the allée effect

3. Interacting Species
    (a) Competition Models
    (b) Predator-Prey Models
    (c) Harvesting effects
4. Combat Models
    (a) Lanchester models of army attrition
    (b) Example of Iwo Jima
5. Relationships
    (a) Romeo-Juliet models
    (b) Marriage/Relationship stability*
6. $F = ma$ (Newton's 2nd law)
    (a) Conservative and nonconservative systems
    (b) Examples
        i. Projectile motion
        ii. Harmonic oscillator (linear spring)
        iii. Nonlinear spring (Duffing equation)
        iv. Pendulum
        v. Sliding bead on a spinning loop
7. Functional optimization*
8. Partial Differential Equations
    (a) Vibrating string under tension (wave equation)*
    (b) Density-dependent transport (e.g. traffic flow)*