AMATH 410
Homework 1

DUE: 8:30 a.m. on Weds., Apr. 16, 2008

Please note: You are encouraged to discuss the homework and work together. However, it is essential that you prepare your own solution writeup and final MATLAB code.

For each problem: together with any analysis or explanations, turn in both all code and all relevant plots, carefully labeled and with all line styles, marker sizes etc. adjusted for maximum readability, to achieve full credit.

I **MATLAB: Resource-limited reproduction:** Recall the resource-limited cell reproduction model discussed in class:

$$n(t) = 4 * n(t-1) * p * (1 - \frac{n(t-1)}{K})$$

Here, each cell divides into 4 copies at each generation, and a fraction $p*(1-n(t-1)/K)$ of these survive. Here, $0 < p < 1$, and K is a critical population size. The interpretation is that survival rate decreases with population size.

We are going to combine this with the Chinook Salmon model given in equation [1.1] of Ellner and Guckenheimer. (Note: this is certainly a real-life example – look at http://wdfw.wa.gov/fish/chum/chum-3e.htm if you'd like!). If we assume that resources are limited that control salmon spawning, we arrive at the model:

$$S(t) = E*(p_4*(S(t-4))+(1-p_4)*(S(t-5)))*(1-\frac{(p_4*(S(t-4))+(1-p_4)*(S(t-5)))}{K});$$

Use initial data `[1005 1241 1520 804 867]` (number of spawners in years `2001, 2002, 2003, 2004, 2005`). Use the parameter value $p_4 = 0.28$, given in Ellner and Guckenheimer; also, take $K = 2000$. For starters, fix $E = 0.57$, as in Ellner and Guckenheimer.

- Write a MATLAB code that simulates this model, and plots numbers of spawners $S(t)$ for years 2006-2306 (300 years). Please save this code as `salmon_nonlinear.m` and hand it in together with the plots requested below. For all plots in this and every assignment, make sure each axis is labeled and the plot is given a clear title.
- Gradually increase E from 0 to 4, running the model to plot $S(t)$ for times from 2006-2306 for each value of E that you select (try at least 10 values of E). Describe in a sentence or two the different **qualitative** types of dynamics than you see as E is varied (growth, decay, anything else)? Use the subplot command (see Lab Manual, exercise 3.3) to print out a single figure that gives four to five plots of $S(t)$, each for a different value of E, that illustrate the main qualitative types of dynamics that you found.

II **MATLAB: Iterating Leslie Matrices and the Euler-Lotkerra Formula.**

- Consider an age-structured population model as follows: maximum age $A = 3$. Also: $p_0 = 0.5$, $p_1 = .9$, $p_2 = .95$, $f_0 = 0$, $f_1 = 1$, $f_2 = 5$, $f_3 = .5$. Write a MATLAB model that simulates the state vector $\mathbf{n(t)}$ of individuals of each age $a = 0, 1, 2, 3$. Start with an initial population consisting of 100 individuals of each age. Plot as functions of time (1) the log of the total population size $N(t) = n_0(t) + n_1(t) + n_2(t) + n_3(t)$

and (2) the fraction of individuals in each age, $w_a(t) = n_a(t)/N(t)$, for $a = 0, 1, 2, 3$. Do this from $t = 1$ to $t = Tmax$, where $Tmax = 50$. Use the polyfit function to fit a first order polynomial to the log $N(t)$ and report the growth rate λ. *Turn in the code you used for this.*

- Write down the Euler-Lotkerra formula for this example, and solve it numerically (use .m files similar to those from class) for the population growth rate λ. How close are your predictions of λ from the Euler-Lotkerra formulas and from the simulations above? *Turn in the code you used for this.*

III **Understanding stage-class models** E+G exercise 2.8. Also, state whether or not the Euler-Lotkerra formula can be used to describe this model in the form that it's given in the text.