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1. Compute the moment generating function for Gaussian random variable from integral

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} e^{-sx} dx.$$

   What is the result if $s = i\omega$?

2. Verify that

$$f(x) = \frac{1}{\sqrt{4\pi Dt}} e^{-(x-Vt)^2/4Dt}$$

   is a solution to the partial differential equation

$$\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial x^2} - V\frac{\partial f}{\partial x}. \tag{1}$$

   [In fact, you should be able to solve Eq. 1 using Laplace transform method and the result in Prob. 1].

3. Try to show the following equality:

$$\int_{-\infty}^{\infty} \left(\frac{1}{\sqrt{4\pi Dt_2}}\right) e^{-\frac{(x-z)^2}{4Dt_2}} \left(\frac{1}{\sqrt{4\pi Dt_1}}\right) e^{-\frac{(z-y)^2}{4Dt_1}} dz = \left(\frac{1}{\sqrt{4\pi D(t_1+t_2)}}\right) e^{-\frac{(x-y)^2}{4D(t_1+t_2)}},$$

   where $t_1, t_2 > 0$. This relation is known as Kolmogorov-Chapman equation; it is a reflection of that a Bownian motion is Markovian.