

Midterm Exam (Open book and note; point values in square brackets)

1. [35] Find the solution to $y'' + \varepsilon y' + \varepsilon y = \varepsilon$, ($x > 0$, $0 < \varepsilon \ll 1$), $y(0) = y'(0) = 0$, through terms of order ε^2 .
2. Consider the ODE $(xy')' + y = 0$. For each of the parts below, justify with an appropriate asymptotic analysis.
[20](a) How many linearly independent solutions of this equation go to zero as $x \rightarrow 0$?
[25](b) How many linearly independent solutions of this equation go to zero as $x \rightarrow +\infty$?
3. Consider the equation $r^2 = e^{\varepsilon r}$, $0 < \varepsilon \ll 1$.
[20](a) Using regular perturbation theory, find two solutions up through $O(\varepsilon)$ terms.

Extra credit [+15]:

- (b) Find the asymptotic form of a third solution for small ε . This is a bit tricky. Five points will be given for the easier task of just showing whether it is negative or positive.