AMATH 581 Autumn Quarter 2007

## Homework 3: Reaction-Diffusion Systems

Consider the $\lambda - \omega$ reaction-diffusion system

$$U_t = \lambda(A)U - \omega(A)V + D_1\nabla^2 U \tag{1a}$$
$$V_t = \omega(A)U + \lambda(A)V + D_2\nabla^2 V \tag{1b}$$

where $A^2 = U^2 + V^2$ and $\nabla^2 = \partial_x^2 + \partial_y^2$.

**Boundary Conditions:** Consider the three boundary conditions in the $x-$ and $y-$directions:

- Periodic
- No flux: $\partial U/\partial n = \partial V/\partial n = 0$ on the boundaries

**Numerical Integration Procedure:** The following numerical integration procedures are to be investigated and compared.

- For the periodic boundaries, transform the right-hand with FFTs
- For the no flux boundaries, use the Chebychev polynomials

You can advance the solution in time using ode45 (or any one of the built in ODE solvers from the MATLAB suite).

**Initial Conditions** Start with spiral initial conditions in $U$ and $V$.

```
[X,Y]=meshgrid(x,y);
m=1; % number of spirals
u=tanh(sqrt(X.^2+Y.^2)).*cos(m*angle(X+i*Y)-(sqrt(X.^2+Y.^2)));
v=tanh(sqrt(X.^2+Y.^2)).*sin(m*angle(X+i*Y)-(sqrt(X.^2+Y.^2)));
```

Consider the specific $\lambda - \omega$ system:

$$\lambda(A) = 1 - A^2 \tag{2a}$$
$$\omega(A) = -\beta A^2 \tag{2b}$$

Look to construct one- and two-armed spirals for this system. Also investigate when the solutions become unstable and "chaotic" in nature. Investigate the system for all three boundary conditions. Note $\beta > 0$ and further consider the diffusion to be not too large, but big enough to kill off Gibbs phenomena at the boundary, i.e. $D_1 = D_2 = 0.1$.