This is not an exhaustive list, but a summary of what you should review or study. The midterm will based on material in Trefethen Lectures 1–17, focusing on material covered in lecture and homework. You may use 2 full sheets (4 fronts and backs) of notes.

Summary of Topics

1. Fundamental concepts from undergraduate linear algebra, such as, linear independence, basis, dimension, range of a matrix and its relation to column space of a matrix, null space of a matrix, rank of a matrix, all matrix-vector multiplication rules, etc. This includes the ability to recognize an outer product as a matrix, and inner product as a scalar.

2. Inner product, orthogonality, adjoint of matrices and vectors (conjugate transpose), rules involving conjugate transposes of products of vectors and matrices. *i.e.* $(AB)^* = B^*A^*$. Understanding the alternative notations based on inner products and matrix vector products. *e.g.* $(x, Ay) = x^*Ay$, or $(x, y) = x^*y$, or $(Ay, x) = (Ay)^*x = y^*A^*x$.

3. Special matrices: know the definition and properties of the following (some of these are related):

   - Unitary matrices.
   - Orthogonal matrices.
   - Hermitian (self-adjoint) matrices.
   - Symmetric matrices
   - Projectors
   - Full rank, vs. rank-deficient matrices.

4. Norms:

   - Definition of p–norms
   - Relation of the 2–norm and inner product.
   - Matrix norms—general and induced matrix norms.

5. The SVD

   - Definition of singular vectors and values.
   - Defining properties of a singular value decomposition vs. simply a product of 3 matrices.
   - Ability to find the SVD of a matrix, if the matrix is simple enough that the singular vectors and values can be determined by inspection.
   - Geometric interpretation of the SVD.
   - Difference between full and reduced SVD.
   - Bases implied by an SVD: basis for the range of A, space orthogonal to the range of A, basis for the null space of A and space orthogonal to the null space of A, all from the SVD of A.
   - Relation of the SVD and rank of a matrix.
   - Relation of the SVD to the condition number of a matrix.
   - Relation of the SVD to the 2–norm of a matrix.
   - Closest rank-deficient approximations to a matrix.

6. Projectors

   - Definition of projectors
   - Geometric understanding of projectors
   - Difference between orthogonal projectors and general projectors (oblique and orthogonal.)
   - Ability to construct projectors onto a space, given vectors that span that space.

7. The QR factorization.

   - Definition of the QR decomposition.
   - Difference between the full and reduced QR
   - Relationship between the QR factorization and algorithms for determining it, such as Gram-Schmidt (classical and modified), and Householder triangularization.
   - Basic idea of Householder triangularization.

8. Least Squares Problems

   - Relationship between least squares problem for data fitting and $Ax = b$.
   - Relevance of the 2–norm in least squares problems.
   - Relevance of the column space of A to the least squares problem.
   - Role of a projector in least squares problems.
   - Definition and properties of the residual in least squares problems.
   - Understanding the pseudoinverse, when A is full rank with $m > n$, but not invertible.
   - Basic idea behind Algorithms 11.1-11.3 for solving Least squares problems.

9. Conditioning

   - Basic idea of the conditioning of a general problem instance.
   - Absolute and relative condition number
   - Condition of problems in linear algebra, and relation to matrix norms.
   - Condition of a matrix, invertible and full column rank.

10. Stability

    - Definition of stability.
    - Definition of backward stability.
    - Relationship between accuracy, conditioning and backward stability.