Appendix B: CMP

Contents

B1 Background and General Impressions

The Connected Mathematics Project (CMP) was developed by Glenda Lappan and others at Michigan State University and funded by the National Science Foundation. The 1998 edition was published by Dale Seymour Publishers. The curriculum was developed to be in line with the pedagogy and content in the 1989 National Council of Teachers of Mathematics (NCTM) standards, namely the *Curriculum and Evaluation Standards for School Mathematics*, the *Professional Standards for Teaching Mathematics*, and the *Assessment Standards for School Mathematics*. More information on CMP can be found at their website (http://www.math.msu.edu/cmp/index.html).

CMP focuses on mathematical content in the number, geometry, measurement, algebra, statistics and probability strands. The "Getting to Know CMP" guide, which comes with each grade's books, stresses that CMP students use the processes of counting, visualizing, comparing, estimating, measuring, modeling, reasoning, playing and using tools. Each of the grade levels has eight modular units. Some of these units have titles such as *How Likely Is It?* (probability) and *Growing, Growing, Growing* (exponential growth) which allow the teacher, students, and parents to get a sense of the mathematical content. A Teacher's Guide and student book is provided for each of the units.

The Teacher's Guides contain all the pages of the student book numbered in a manner consistent with the teacher pages, a "Teaching the Investigation" section, different types of assessments, additional problems, samples of student work, articulation information for the teacher, blackline masters, and form letters to parents in both English and Spanish describing the purpose of each unit and how they can best support their child's mathematical development at home. The "Teaching the Investigation" sections are the heart of the CMP curriculum. They give the teacher guidance on how to teach the lesson, an explanation of the mathematics in the lesson, and specific questions to ask students to make sure the important mathematical points are brought out during class. In the absence of a mentor teacher in each building, the assistance provided by the "Teaching the Investigation" sections could be very valuable to teachers that are not yet comfortable with the mathematics or with the discovery method of teaching. Even though CMP provides enough guidance to support a novice teacher, an experienced teacher can use his or her own creativity to supplement lessons and to meet the individual needs of students.

A CMP lesson, called an *Investigation*, is organized in three parts: *Launch*, *Explore*, and *Summarize*. *Launch* is the lesson introduction; it includes definitions, explanation of relevant concepts and other background material. *Explore* encourages students to work individually, then in pairs or groups, while the teacher circulates through the classroom and listens to students. *Summarize* allows for groups to share their findings followed by a class discussion. The typical lesson concludes with a "Mathematical Reflections" section, enabling students to reflect on their own learning. CMP emphasizes

a discovery-based approach to learning that encourages students to select, adapt, and analyze problem solving strategies in order to develop mathematical understanding and become autonomous learners.

The curriculum provides numerous projects and problems for students. "ACE" problems appear at the end of each Investigation. These are grouped into Applications, Connections and Extensions. The Application problems reinforce the ideas currently being studied. The Connection problems integrate these ideas with strands that have been previously taught. The Extension problems are often the most challenging and carry the ideas forward. The Teacher's Guides also contain a *Question Bank* with additional problems that can be assigned to students that require more practice. Most of the twenty-four books in the curriculum have a Unit Project that requires the students to use the mathematics they have learned in pursuit of a larger goal. Students also have journals in which they can write their thoughts and record their work. The curriculum suggests that teachers should check these journals for completeness and not for correctness, so students are free to express their thinking. By examining these records, both correct and incorrect, the teacher is better able to assist students and assess the progress of his or her class.

The curriculum is constructed around five instructional themes: teaching for understanding ("big ideas"), connections, investigations, representations and technology. For example, CMP presents mathematics in a coherent way with an emphasis on connections among the mathematical ideas - thus the title of the curriculum. Students are urged to use multiple representations. For example, in the section *Solving Linear Equations* from the 8^{th} grade book *Say It With Symbols*, students are asked to compare graphical, tabular, and symbolic representations of a linear function. Regarding the technology theme, CMP 6^{th} grade students are required to have a standard "four-function" calculator and 7^{th} and 8^{th} grade students utilize graphing calculators with statistical capabilities. Some computer programs are included with the curriculum for enhancing probability and geometry lessons. We find that in most cases these technologies did not replace pencil and paper arithmetic, but since students themselves choose when to use calculators, a dependence on the calculator for problems of too low a level could develop.

Turning to the mathematical content of CMP, we find that most of the concepts presented in the number strand are a review for students that have gone through, for example, the Everyday Mathematics curriculum for grades K-6. The number strand is arguably the most basic and fundamental mathematics strand and much of the presentation in CMP is below the level articulated in the 2000 NCTM number standard for grades 6-8. Specifically, we find that CMP students are not expected to compute fluently, flexibly and efficiently with fractions, decimals and percents as late as 8^{th} grade. Standard algorithms for computations with fractions (e.g. $\frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$, $\frac{a}{b} \div \frac{a}{c} = \frac{c}{b}$) are often not used. We understand that the developers of CMP are aware of the absence of material on division of fractions and probably will correct this in the next edition. Conversion of fractions to decimals is discussed only in simple cases such as for fractions

with denominators of ten, and CMP lacks a discussion of repeated decimals. A discussion of long division is also missing. Such a discussion could make the conversion of fractions like $\frac{1}{7}$ to decimal form a simple procedure and would tie in with a discussion of rational numbers and repeated decimals. Long division is also a basis for the division of algebraic polynomials that students will see in high school. Multiplication of fractions is discussed in 7^{th} grade but mostly in simple cases. This is an area where multiplication algorithms could be exploited to solidify the concept of place value.

CMP does a good job of helping students discover the mathematical connections and patterns in the algebra strand, but falls short in a follow-through with more substantial statements, generalizations, formulas or algorithms. For example, in *Growing, Growing, Growing* exponents are discussed, but the exponential laws are not explicitly written down for the students even after they are discovered. In one exercise students discover that $2^6 = (2^2)^3$, but they need more practice to reach the generalization that $(a^n)^m = a^{nm}$. There is no discussion of negative and fractional exponents except when students explore exponential functions using graphing calculators. As a result, students miss an opportunity to revisit square roots and cube roots. In the 8^{th} grade unit *Frogs, Fleas and Painted Cubes*, students are required to be able to recognize that the same equation can be modeled in more than one way, but CMP misses the opportunity to discuss the quadratic formula or the process of completing the square.

Many mathematicians and educators believe that when using a curriculum that relies on discovery learning, such as CMP, teachers must understand the material even better than when teaching from a more traditional curriculum. Moreover, since students often work in pairs or in groups, teachers must be effective in establishing a classroom where all students participate in the mathematical work. Also, in order for students to effectively discover the mathematics, more time needs to be devoted to the lessons than in a traditional curriculum. The recommended minimum of 45 minute-long classes seems insufficient.

In conclusion, CMP corresponds well to the 2000 NCTM Principles and Standards, with the notable exception of the number standard. We feel that CMP's overwhelming emphasis on conceptual development neglects standard computational methods and techniques. In our opinion, concepts and computations often positively reinforce one another. While we understand that CMP seems to be motivated by the criticism that traditional curricula produce students that can compute but lack conceptual understanding, there is a danger here of producing students with conceptual understanding but limited computational skills. CMP admits that "because the curriculum does not emphasize arithmetic computations done by hand, some CMP students may not do as well on parts of the standardized tests assessing computational skills as students in classes that spend most of their time on practicing such skills." This statement implies we have still not achieved a balance between teaching fundamental ideas and computational methods.

B2 Methodology

In the remaining sections, we use the 2000 NCTM Principles and Standards as a guide to gather information about the CMP curriculum. We then use this information (evidence) to score the curriculum on how well it compares with these standards and principles. A brief summary is given for each of the ten overarching standards. A brief summary is also given for the principles. We now describe the source of our questions, the meaning of our scoring system, the criteria we use in presenting the evidence, and the purpose of the brief summaries.

The Questions:

The questions for each of the ten overarching 2000 NCTM Standards were taken verbatim from the "bullets" listed as the *Expectations* for what grades 6-8 students should be able to do. These can be found in the *Principles and Standards for School Mathematics* section of the NCTM web page, http://www.nctm.org. We formulated each bullet as a question, and as a result, the number of questions we examine is not the same for each overarching standard. The overarching standards and the number of questions used for each are: Number(14), Algebra(9), Geometry(12), Measurement(9), Data Analysis and Probability(10), Problem Solving(4), Reasoning and Proof(4), Communication(4), Connections(3), and Representation(3).

Most of the questions for each of the six 2000 NCTM Principles were taken from the *Guidebook to Examine School Curricula – TIMMS as a Starting Point to Examine Curricula, Attaining Excellence Report, US Dept. of Education, Office of Educational Research and Improvement.* This report is available at http://timss.enc.org/topics/timss/kit. The project team composed other questions after reading in detail the *Principles for School Mathematics* document, available at the NCTM site. These questions were posed in the same format as those from the *Guidebook* above. These principles and the number of questions used for each are: Equity(4), Curriculum(3), Teaching(1), Learning(2), Assessment(2), and Technology(1).

The reasons we chose the 2000 NCTM Principles and Standards as our guide are threefold. First, these standards are the most recent version available from NCTM. The second reason is the availability and knowledge of these standards by the various communities that may benefit from our report. The third reason is that these standards represent a tremendous amount of thoughtful effort from a community with a wide variety of expertise, and we can comfortably subscribe to them, whether or not we agree with them in every detail.

The Evidence:

Each book involved in the study was examined and references gathered to compare the curricular materials with the standard. In reporting this evidence, we seek to meet the following two goals:

1. List as much evidence as necessary to support our conclusion about how fully the standard is met.

2. List evidence for each grade to show how well the curriculum builds up the concepts across the grades to meet the standard. Our goal is to list evidence at all three grades *(6)*, *(7)*, and *(8)*.

With these goals in mind, it is important to know the following before reading the report:

1. For the Number, Algebra, Geometry, Measurement, and Data and Probability Standards, we are evaluating whether the entire middle grades curriculum meets the standard. Every grade level does not necessarily have to address every question. The 2000 NCTM Principles and Standards document makes this point clear. If the standard is met before 6th grade, we note that as well.

2. For the Problem Solving, Reasoning and Proof, Communication, Connections, and Representation Standards, and the six Principles, we are evaluating whether the standard (or principle) is frequently met. The 2000 NCTM Principles and Standards document also makes this point clear. The expectation is that evidence of the standard (or principle) should be seen frequently in each grade.

3. If evidence is not supplied for a grade level, it is because we were not able to find any.

4. In answering different questions, we may supply different amounts of evidence. It is possible that one of the first five standards is met totally in one book at one grade level. If this happens, we still make an effort to report what has happened at grades below or above to show the entire conceptual story across the grades in the study. On the other hand, if the materials only meet part of the standard, we still report the conceptual story that is found in the materials.

The Scoring:

The questions relating to the ten NCTM overarching standards were scored with the numbers 3, 2, 1, and 0. The meaning of each is given below.

- **Score of 3: The standard is fully met.** By the end of the *(8)* books, the answer to the question is an unqualified yes.

- **Score of 2: The standard is adequately met.** By the end of the *(8)* books, the answer to the question is not an unqualified yes. Evidence has been found for meeting parts of the standard. A judgment has been made that the curriculum's material, though not fully meeting the standard, is adequate for use.

- **Score of 1: The standard is not adequately met.** By the end of the *(8)* books, the answer to the question is not an unqualified yes (same as for a score

of 2). Although evidence has been found for meeting parts of the standard, a judgment has been made that the curriculum's material related to this standard is not adequate for use.

- **Score of 0: The standard is not met.** By the end of the *(8)* books, no evidence was found for the standard.

The questions relating to the six 2000 NCTM Principles are also scored with the numbers 3, 2, 1, and 0. The meaning for each of these scores is given directly below each question. The score of 2 also carries the connotation that the material does not fully meet the standard, but is adequate for use, while a score of 1 implies that the material related to this standard is not adequate for use.

The Summaries:

After each overarching standard and the principles are scored, we tabulate the scores for each question. We then give a short written summary. The purposes of this written summary are twofold:

1. To highlight the main deficiencies that led to the scores less than 3.
2. To point out, in some cases, facts worth mentioning that the evidence revealed, but that the scoring did not reflect.

B3 Number Standard

B3.1 Number Questions and Evidence

Note: In what follows IV1 refers to Investigation 1, IV2 to Investigation 2, etc.

1. *Does the curriculum enable all students to work flexibly with fractions, decimals, and percents to solve problems?*

   Score: 2

   Evidence:

   - *(6) Bits and Pieces I.* This is a beginning unit focused on developing the meaning of fractions, decimals, and percents and in which situations to apply each. Students are not expected to compute flexibly with fractions, decimals, and percents yet. There is more emphasis on deciding *when* it is appropriate to represent information using each. The extent of solving problems is dividing a rectangular pan of brownies into 20 pieces.
   - *(6) Bits and Pieces II.* In IV4, students add and subtract fractions with like and unlike denominators for simple cases. They develop their own algorithm and on p. 48 are asked to explain how it works. In IV1 and IV2, students solve problems with sales tax, tips, and discounts. They really don't need to know how to divide since problems are cast as multiplication ones. For example, given the sales price and the percentage discount, the original price is not to be calculated. Instead the sales price is asked for, given the percentage discount and the original price. In IV5, students multiply fractions using the area model, and in doing so discover the standard algorithm. In IV6, students add, subtract, and multiply decimal numbers, but do not divide them.
   - *(7) Comparing and Scaling.* In IV2 and IV3, comparisons are made using percents and ratios, respectively.
   - *(8) Looking for Pythagoras.* Irrational numbers are introduced. Students change repeating decimals to fractions on p. 56.

   Discussion: This standard is adequately met. According to the standards, students should have learned how to recognize equivalent forms of fractions, decimals, and percents in the benchmark cases in grades 3-5 and more flexibility and fluency is expected in grades 6-8. The discussion of this standard for grades 6-8 mentions that students should be *facile* in working with fractions, decimals and percents. These units adequately meet this standard by setting a firm conceptual foundation, but for grades 6-8, much more facility should be expected. There is no evidence that by the end of 8th grade students could divide 23/45 by 1/6, multiply .43 by .27, or write (1+4/5)/9 as a single fraction.

2. *Does the curriculum enable all students to compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line?*

   Score: 3

   Evidence:

   - *(6) Bits and Pieces I.* Students are beginning the study of comparisons with fractions, decimals, and percents. They are not efficient in their use yet. No algorithms are analyzed yet. On p. 23, students compare fractions by making fraction strips and locate fractions on a number line using these strips. The treatment is concrete to build a firm base of understanding. On p. 23, they compare fractions to a benchmark fraction. They aren't required to compare them in general without using a calculator. In IV4, they order and compare decimals to place value 10^{-4}. On p. 58, prob 3, students estimate a "good" fraction to represent a decimal. On p. 81 they locate fractions, decimals, and percents on a number line.
   - *(6) Bits and Pieces II.* In IV3, students learn fraction benchmarks. On p. 33, they locate fractions on a number line in relation to these benchmarks. On p. 36, problem 36, they order decimals and on p. 74, problem 11, they locate decimals on a number line.
   - *(7) Comparing and Scaling.* In IV2 and IV3, comparisons are made using percents and ratios, respectively.
   - *(8) Looking for Pythagoras.* Students compare the square root of 10 to the nearest fraction on p. 63.

   Discussion: This standard is fully met. We note that students are not comparing general fractions by finding the common denominator with pencil and paper without the aid of a calculator. The score of 3 reflects our interpretation that the NCTM Standards say a calculator is a valid tool for accomplishing these tasks for more general fractions.

3. *Does the curriculum enable all students to develop meaning for percents greater than 100 and less than 1?*

   Score: 3

   Evidence:

   - *(6) Bits and Pieces I.* On p. 74, students work with 12.5% discount. On p. 79, 1/2 of a square on a 100 grid is shaded to show a percent less than 1. On p. 11, the terminology "exceeding the goal" was used and the fraction $1\frac{1}{4}$ was used.
   - *(6) Bits and Pieces II.* On p. 13, problem 14, students find 3 ways to represent 120% of a dollar.

- *(7) Data Around Us.* On p. 51, 1178% was used when talking about exponential growth problems over time.

Discussion: This standard is fully met.

4. *Does the curriculum enable all students to understand and use ratios and proportions to represent quantitative relationships?*

   Score: 3

   Evidence:

   - *(7) Stretching and Shrinking.* In IV5, students find the missing side lengths in similar triangles.
   - *(7) Comparing and Scaling.* In IV3, p. 27, students use part-to-part and part-to-whole comparisons. In IV4, they compare by finding rates such as mi/gal, $dollars/hr$, and use proportional reasoning with these unit rates to solve problems. They use rates and proportions with understanding.

   Discussion: This standard is fully met.

5. *Does the curriculum enable all students to develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation?*

   Score: 3

   Evidence:

   - *(6) Prime Time.* On p. 49, exponential notation with prime numbers is given, e.g. 5^4.
   - *(7) Data Around Us.* Students use the contexts of natural disasters (exponential growth problems) to study large numbers. These numbers are compared in IV3 and rounded on p. 28. On p. 41, scientific and calculator notations are introduced. Students work problems using these notations. On p. 45, connections between prime factorization and exponential notation are made.
   - *(8) Growing, Growing, Growing.* In IV1, functions of the form $y = a^x$ are studied. In IV4, exponential decay of the form $y = a^{-x}$ is studied. Both exponential growth and decay are seen from tables, graphs and equations. Graphing calculators are used on p. 50.

   Discussion: This standard is fully met.

6. *Does the curriculum enable all students to use factors, multiples, prime factorization, and relatively prime numbers to solve problems?*

   Score: 3

   Evidence:

- *(6) Prime Time.* The entire unit is about factors, multiples, prime factorization, and relatively prime numbers. In IV1, computing factors is practiced using the Factor game. In IV2, multiples are studied. In IV3, factor pairs are studied. In IV4, students find common factors and multiples. In IV5 and IV6 students use the Prime Factorization Theorem. On p. 48, students discover this theorem. On p. 51, they work with relatively prime numbers. In IV6, the popular locker problem (the locker doors are opened and closed in particular patterns related to factors and primes) is solved using the concepts developed in the unit. Students work with the GCF and the LCM on p. 50.

Discussion: This standard is fully met.

7. *Does the curriculum enable all students to develop meaning for integers and represent and compare quantities with them?*

   Score: 3

   Evidence:

   - *(7) Accentuate the Negative.* Understanding of addition, subtraction, multiplication and division with negative and positive integers is developed. This understanding is developed using the number line, red and black chips, games in the contexts of negative temperatures, bank accounts, profit, and deficit.
   - *(7) Data About Us.* Understanding is further developed for large positive integers by measuring them in terms of benchmarks that have meaning to students.

   Discussion: This standard is fully met.

8. *Does the curriculum enable all students to understand meaning and effects of arithmetic operations with fractions, decimals, and integers?*

   Score: 1

   Evidence:

   - *(6) Bits and Pieces I.* Only the simpliest arithmetic is used.
   - *(7) Bits and Pieces II.* Students add, subtract and multiply fractions. They multiply decimals by whole numbers. No division is done with fractions and decimals. The curriculum does not include division of fractions.
   - *(7) Accentuate the Negative.* Students do operations with addition, subtraction, multiplication, and division of integers. Division is just seen as the inverse of multiplication and problems are cast that way. The complexity of the problems is of the level $-3.4+? = -5.6$.

   Discussion: This standard is not adequately met. The arithmetic operations of dividing fractions, multiplying decimal numbers, or multiplying general fractions

are studied only for simple cases and not to an extent that would ensure students understand the "meaning and effects" of these operations. Specifically, the meaning of division is not dealt with in enough detail. It is conceivable that students can complete a middle-grades mathematics program without having seen the effects of dividing fractions and decimals.

9. *Does the curriculum enable all students to use the associative, commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals?*

   Score: 1

   Evidence:

   - *(6) Bits and Pieces II.* There are no computations complicated enough that simplification is needed.
   - *(7) All Units.* There are no computations complicated enough that simplification is needed.
   - *(8) Frogs, Fleas, and Painted Cubes.* On p. 84f, the teacher is told what the order of operations are. On p. 39, the factoring of quadratic equations leads to a discussion of the distributive property. This is the first such discussion in the curriculum.
   - *(8) Say It With Symbols.* IV1 is entitled "Order of Operations." On p. 34, students see the distributive property and on p. 35 the commutative property of addition and multiplication. On p. 45, students must insert parentheses to make the statement a valid equation. This is building toward the associative property, but this property is never mentioned.
   - *(8) Kaleidoscopes, Hubcaps, and Mirrors.* In IV4, the commutative property of addition is given for numbers in order that a discussion can go forward about whether there is a commutative property for symmetry rotations.

   Discussion: This standard is not adequately met. The associative property is not mentioned.

10. *Does the curriculum enable all students to understand the use of inverse relationships of addition and subtraction, multiplication and division, and squaring and finding the square roots to simplify computations and solve problems?*

    Score: 3

    Evidence:

    - *(6) Bits and Pieces II.* The inverse relationship of addition and subtraction is made clear. Division of fractions and decimals are not present.
    - *(7) Accentuate the Negative.* In IV3, subtraction is seen explicitly as the inverse of addition. In IV4, p. 59, division is presented as "undoing" multiplication.

- *(8) Looking for Pythagoras.* On p. 19, square root is introduced. In IV4, students use the Pythagorean Theorem to solve problems using right triangles.

Discussion: This standard is fully met.

11. *Does the curriculum enable all students to select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply selected methods?*

Score: 3

Evidence:

- *(6) Bits and Pieces I.* Students use benchmark fractions to compare and estimate values of other fractions. Not much computation is done, rather a firm understanding of the concepts is built here. For example, in IV4, students try to figure out how to add fractions with unlike denominators before any algorithms are given. This gives them a chance to struggle with the concepts. On p. 58, prob 3, students estimate a "good" fraction to represent a given decimal number.
- *(6) Bits and Pieces II.* Students estimate by comparing to benchmarks in IV3. On p. 37, estimation is done without the aid of a calculator. On p. 24, students do 15 easy problems involving percents. For these problems, they are allowed to use a calculator, but only a few should require one. For example 5% of 40 could be done without a calculator. (The teacher must be careful that an over-dependence on the calculator is not developed and that mental strategies do not suffer.)
- *(7) Data Around Us.* On p. 47, mental calculation of $15{,}000 \times 50{,}000$ is done.

Discussion: This standard is fully met.

12. *Does the curriculum enable all students to develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use?*

Score: 1

Evidence:

- *(6) Bits and Pieces I.* Students find equivalent fractions in simple cases. They convert decimals to fractions (out of 100). They are not yet fluent in computing with fractions and decimals as evidenced by the simplicity of the problems they do.
- *(6) Bits and Pieces II.* Students do develop strategies and probably arrive at the standard ones. They must explain to the teacher and to other students their strategy and test it out on several problems. The class discusses the strategies to make sure they are correct. The missing component here is is

the fluency. They can not divide fractions yet, or decimals, or work with general numbers away from their benchmarks.

- *(7) All units.* Very few problems require more than simple computation. Not enough problems are included that require the use of the associative, commutative, and distributive properties.
- *(8) All units.* No associative property is evident. No division of fractions is found. Not many simplification strategies are needed that would actually test the conceptual knowledge of the properties above. Not much fluency of use is required.

Discussion: This standard is not adequately met. Students develop and analyze algorithms, but they are not fluent in the use of these algorithms except in simple cases. Not enough work is done with multiplying decimal numbers (say .23x.37) from even a conceptual point of view that reinforces place value and use of the distributive law. These units are at a lower mathematical level than the standards say should be present in grades 6-8.

13. *Does the curriculum enable all students to develop and use strategies to estimate the results of rational number computations and judge reasonableness of results?*

Score: 3

Evidence:

- *(6) Bits and Pieces II.* In IV3, students estimate the results of fraction and decimal computations. On p. 42, the reasonableness is judged using the vicinity of a benchmark. No instructions are given in the student manual about asking if it seems reasonable other than comparing to a benchmark. On p. 42a, the quote "The primary objective is for them to use benchmarks for making sense of fractions and decimals as quantitites." On p. 42b, benchmarks are refined to more decimal places.
- *(7) Comparing and Scaling.* On p. 46 students are asked, "How confident are you that your estimate is accurate? Explain."
- *(7) Stretching and Shrinking.* In problem 4.1 on p. 42, students must say if their estimate is an overestimate or an underestimate. On p. 62 they are asked to explain if they think their height estimates are reasonable.

Discussion: This standard is fully met.

14. *Does the curriculum enable all students to develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios?*

Score: 3

Evidence:

- *(6) Covering and Surrounding.* Students use the idea of scale to design a park based on area specifications.
- *(7) Stretching and Shrinking.* The entire unit is about scaling 2D figures to produce similar figures. The idea of proportion is used to find the missing side lengths in similar triangles.
- *(7) Comparing and Scaling.* On p. 40, problem 4.2b, students use the rate table to solve problems using proportions.

Discussion: This standard is fully met.

B3.2 Number Summary

Question	CMP
Number 1.	2
Number 2.	3
Number 3.	3
Number 4.	3
Number 5.	3
Number 6.	3
Number 7.	3
Number 8.	1
Number 9.	1
Number 10.	3
Number 11.	3
Number 12.	1
Number 13.	3
Number 14.	3

Table 1: Summary of NCTM Number and Operations Standard Results

The scores in the table above show that the curriculum fully meets ten of the standards, adequately meets one of them, and does not adequately meet three of them. The lower scores were given because the curriculum does not enable students to adequately work fluently with fractions, decimals, percents, and integers at the mathematical level expected for grades 6-8. Furthermore, the curriculum does not address the associative law. The evidence points to several other issues worth mentioning that the scoring did not reflect.

- Negative numbers are not studied until the 7th grade.
- The associative, distributive, and commutative laws are not encountered until the 8th grade. Moreover, these concepts do not build up from early arithmetic, which would make the algebraic notions stronger.
- Proportionality and ratios are not studied until 7th grade.

B4 Algebra Standard

B4.1 Algebra Questions and Evidence

Note: In what follows IV1 refers to Investigation 1, IV2 to Investigation 2, etc.

1. *Does the curriculum enable all students to represent, analyze, and generalize a variety of patterns with tables, graphs, words, and when possible symbolic rules?*

   Score: 3

   Evidence:

   - *(6) Prime Time.* Patterns are introduced for square numbers and rectangular numbers on p. 43.
   - *(7) Variables and Patterns.* In IV1, students collect data, and make tables and graphs for the same relationship. In IV2, p. 25, they analyze what could be true for points between the data points in the graph (getting to the idea of interpolation). Students go between words, tables, graphs, and simple symbolic equations to describe patterns in the context of a bicycle tour business. They look for a pattern in a table, fill in a table from a graph, write words and a formula. On p. 37 two bike companies send out information and one is in tabular form and one in graphical form. The students must make a decision which company has the better deals. On p. 39, they extrapolate to predict future profit from a graph.
   - *(7) Moving Straight Ahead.* In IV1, students predict how the pattern will continue. They determine the two variables that are involved and state their relationship.
   - *(8) Growing, Growing, Growing.* On p. 24, students state whether the pattern in the table is linear or exponential or neither. They write the equation, and draw the graph.
   - *(8) Frogs, Fleas, and Painted Cubes.* In IV3, patterns that are quadratic are investigated, such as triangular and square numbers.
   - *(8) Say It With Symbols.* In IV5, students generalize a pattern to a formula for the surface area of n staggered rectangular prisms as a function of n.

   Discussion: This standard is fully met.

2. *Does the curriculum enable all students to relate and compare different forms of representation for a relationship?*

   Score: 3

   Evidence:

- *(7) Variables and Patterns.* On p. 35 in the Reflection, students evaluate the advantages and disadvantages of using words, formulas, tables, and graphs to show the same relationship.
- *(7) Moving Straight Ahead.* The equation $y = mx + b$ is graphed. Values from the table are found on the graph.
- *(8) Thinking With Mathematical Models.* Tables, equations, and graphs are made for straight lines and nonlinear models.
- *(8) Growing, Growing, Growing.* Tables, equations, and graphs are made to examine exponential growth and decay.
- *(8) Frogs, Fleas, and Painted Cubes.* On p. 17, tables, graphs, and equations are used to describe quadratic relationships. The Arch in St. Louis is given as an example of a parabola on p. 7.

Discussion: This standard is fully met.

3. *Does the curriculum enable all students to identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations?*

Score: 3

Evidence:

- *(7) Variables and Patterns.* On p. 60i in the Teacher's Guide, plots of inverse relationships such as $t = 40/r$ are discussed. These equations are said to be nonlinear as opposed to linear.
- *(7) Moving Straight Ahead.* On p. 34 in the Reflection, students are asked "How do you decide whether a relationship is linear?" Then they say how to compare rates for a linear relationship. They find the y-intercept from the graph and the tabular data, and the equation. On p. 89, problem 23, students have to tell which graphs are linear or nonlinear.
- *(8) Thinking With Mathematical Models.* In IV2, students graph nonlinear relationships. On p. 34, they study equations like $y = a + 2/x$ with graphic calculators. They look at a large number of graphs of both linear and nonlinear phenonemon. On p. 45, four graphs showing biological information about a caribou population are interpreted. On p. 21, students say whether data in a table represents a linear relationship.
- *(8) Growing, Growing, Growing.* On p. 24, students explain whether data in a table are linear or exponential or neither.
- *(8) Frogs, Fleas, and Painted Cubes.* On p. 18g, tables of linear, exponential, and quadratic functions are compared for the teacher.

Discussion: This standard is fully met.

4. *Does the curriculum enable all students to develop an initial conceptual understanding of different uses of variables?*

   Score: 3

   Evidence:

   - *(7) Variables and Patterns.* In IV1, students begin to understand that variables represent amounts of a quantity that changes, and begin to identify two variables to make a graph. The variables are used on the axis of a graph or in symbolic form.
   - *(7) Moving Straight Ahead.* Variables are related to coordinates.
   - *(8) Frogs, Fleas, and Painted Cubes.* In IV3, variables represent the general case in patterns.

   Discussion: This standard is fully met.

5. *Does the curriculum enable all students to explore relationships between symbolic expressions and graphs of lines, paying particular attention to meaning of slope and intercept?*

   Score: 3

   Evidence:

   - *(7) Moving Straight Ahead.* In IV5, slope and y-intercept are formally introduced. Each is seen from the equation, the graph, and the table of data.
   - *(8) Thinking With Mathematical Models.* On p. 13, students find the equation of a line given the slope and one point on the line. On p. 18, they find the equation of the line given two points on the line.
   - *(8) Looking for Pythagoras.* In IV6, students use the slopes of two lines to test whether they are parallel or perpendicular.

   Discussion: This standard is fully met.

6. *Does the curriculum enable all students to use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships?*

   Score: 3

   Evidence:

   - *(7) Variables and Patterns.* Students write a relationship between two variables symbolically. No equations are solved yet.
   - *(7) Moving Straight Ahead.* On p. 54, students write an equation from a story problem that has one linear equation and one variable and find its solution. On p. 63 in the Reflection, students are asked "How do you solve $y = mx + b$ for x if you know y?"

- *(8) Frogs, Fleas, and Painted Cubes.* On p. 39, students match a quadratic equation with its graph. They solve for the x-intercepts by checking in a data table or looking at the graph.
- *(8) Say It With Symbols.* In IV4, p. 53, students solve two equations in two variables symbolically. In IV4, p. 57, equations of the complexity $x^2 + 5x = 0$ are solved for x by factoring.

Discussion: This standard is fully met.

7. *Does the curriculum enable all students to recognize and generate equivalent forms for simple algebraic expressions and solve linear equations?*

   Score: 3

   Evidence:

   - *(7) Moving Straight Ahead.* In IV4, a single linear equation is solved by converting to a simpler equivalent equation by symbolic manipulation.
   - *(8) Say It With Symbols.* On p. 53, students solve two linear equations in two variables by generating equivalent equations.

   Discussion: This standard is fully met.

8. *Does the curriculum enable all students to model and solve contextualized problems using various representations such as graphs, tables, and equations?*

   Score: 3

   Evidence:

   - *(7) Variables and Patterns.* Students are just starting to see what graphs, tables and symbolic formulas are. They are not yet solving equations.
   - *(7) Moving Straight Ahead.* Simple linear equations are written for story problems such as installment paying and forensic science, and are solved using the representations above.
   - *(8) Growing, Growing, Growing.* The Unit Project is to model the half-life of iodine-124 by using sampling techniques to determine the amount that decays at a given time. Students are told to mark one face of a number cube. They draw number cubes from a hat to simulate the fraction of the iodine that decays at each step. They must come up with the equation that models the situation, and make tables and graphs to show their results.

   Discussion: This standard is fully met.

9. *Does the curriculum enable all students to use graphs to analyze the nature of changes in quantities in linear relationships?*

   Score: 3

   Evidence:

- *(6) Covering and Surrounding.* Students make graphs of the diameter versus the circumference of a circle on p. 71.
- *(7) Moving Straight Ahead.* In problem 2.2, p. 18, students are asked how the rate (slope) affects the graph. In problem 5c, p. 26, students compare what graphs would look like based on tabular values. In problem 18c, p. 30, given the graph, students write the equation and generate a table. On p. 34 in the Reflection, students write about how to compare rates for two linear relationships from their graphs.
- *(8) Thinking With Mathematical Models.* On p. 25 in the Reflection, if the slope m is negative, students are asked what happens to y as x increases. In IV4, students sketch graphs that fit written descriptions.

Discussion: This standard is fully met.

B4.2 Algebra Summary

Question	CMP
Algebra 1.	3
Algebra 2.	3
Algebra 3.	3
Algebra 4.	3
Algebra 5.	3
Algebra 6.	3
Algebra 7.	3
Algebra 8.	3
Algebra 9.	3

Table 2: Summary of NCTM Algebra Standard Results

The scores in the table above show that the curriculum fully meets all nine algebra standards. The evidence points to one issue worth mentioning that the scoring did not reflect.

- CMP only includes very minimal algebraic material in its 6th grade curriculum. (Reported in questions 1 and 9.)

B5 Geometry Standard

B5.1 Geometry Questions and Evidence

Note: In what follows IV1 refers to Investigation 1, IV2 to Investigation 2, etc.

1. *Does the curriculum enable all students to precisely describe, classify, and understand relationships among types of 2D and 3D objects (e.g. angles, triangles, quadrilaterals, cyclinder, cones) using their defining properties?*

   Score: 3

   Evidence:

   - *(6) Shapes and Designs.* Students begin by recognizing angles, triangles, quadrilaterals, and regular and irregular polygons. On p. 45, equilateral and isosceles triangles are defined and on page 51 rectangles are defined.
   - *(7) Filling and Wrapping.* The definition of a general right prism is given on p. 26. In IV4 and IV5, cylinders, cones, and spheres are defined. The relationship of cones and spheres to cylinders is given. Students build these and fit them into a cylinder.
   - *(8) Looking for Pythagoras.* In Summarize, p. 16f, the teacher puts 2D figures on the board and students classify them depending on properties such as perpendicular sides, parallel sides, etc.
   - *(8) Frogs, Fleas, and Painted Cubes.* On p. 28, the parabola is studied in terms of its properties (*max* or *min*, x-intercepts, and line of symmetry).

   Discussion: This standard is fully met.

2. *Does the curriculum enable all students to understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects?*

   Score: 3

   Evidence:

   - *(6) Covering and Surrounding.* Students investigate perimeter and area of rectangles and parallelograms. They change the perimeter while keeping the area constant and vice versa. They build their own knowledge of the area of triangles, parallelograms, and trapezoids by constructing them on centimeter grid paper. They also compute the circumference and area of circles.
   - *(7) Stretching and Shrinking.* In IV1, students draw similar figures. In IV2, they use the coordinate system to draw figures called Wumps that are mostly similar. They measure them to see the length scale and area scale for the similar ones. In IV3, the refining of figures to produce smaller congruent figures (called rep-tiles) allows students to study how areas of similar figures

scale. This refining process is extremely useful in solving advanced practical problems and is used in developing computer graphics applications. In IV4 and IV5, the missing sides of similar figures are found.

- *(7) Filling and Wrapping.* In IV6, boxes are scaled up and down to create similar rectangular prisms. The scale factor is calculated and the volumes compared.

Discussion: This standard is fully met.

3. *Does the curriculum enable all students to create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity and Pythagorean relationship?*

Score: 3

Evidence:

- *(7) Stretching and Shrinking.* On p. 54, students determine whether triangles are similar and if so give a scale factor based on the lengths of the sides. In the Reflection, on p. 40, students are asked, "How can you decide whether two figures are similar?"
- *(7) Filling and Wrapping.* In IV3, students are discovering an inductive argument for the volume of any right prism. In problem 18, p. 35, students explain how to find the volume of a right prism with an irregularly shaped base.
- *(8) Looking for Pythagoras.* In IV3, p. 27, the Pythagorean Theorem is introduced. Students make a table with the lengths of the sides and their squares, and use this to conjecture about their relationship based on the pattern seen. On p. 29, a definition of *theorem* is given. Students use geometric ideas to prove the Phythagorean Theorem. On p. 40g, an algebraic proof based on a geometrical proof is given to the teacher.
- *(8) Hubcaps and Kaleidoscopes.* On p. 49, problem 2b, students are asked to find a transformation that moves one circle to another to check for congruence. On p. 51, problem 5 asks the same question for congruence of triangles.
- *(8) Hubcaps and Kaleidoscopes.* In problem 27, p. 57, students are asked, "Investigate what happens when you rotate a figure 180 degrees about a point and then rotate the image 180 degrees about a different point. Is the combination of the two rotations equivalent to a single transformation? Test several cases, and make a conjecture about the result." This is an example of inductive reasoning.

Discussion: This standard is fully met.

4. *Does the curriculum enable all students to use coordinate geometry to represent and examine the properties of geometric shapes?*

Score: 3

Evidence:

- *(6) Shapes and Designs.* On p. 60, problems 12-16, students figure out what four coordinates to plot to make a rectangle, and a parallelogram that is not a rectangle. They also plot triangles.
- *(7) Stretching and Shrinking.* In IV2, coordinates of a Wump are plotted. These coordinates are scaled and a new Wump is plotted. The similarity is investigated. On p. 27d, the teacher is told the rule that (x, y) maps to (nx, ny) is what produced the figures.
- *(8) Looking for Pythagoras.* On p. 15, students investigate the intersection of diagonals of squares, rectangles, and rhombuses to see that they are perpendicular. The lengths of the diagonals are also calculated.
- *(8) Frogs, Fleas, and Painted Cubes.* In IV2, students plot graphs of parabolas.
- *(8) Kaleidoscopes, Hubcaps, and Mirrors.* In IV3, images under symmetric transformations such as reflections, translations, and rotations are examined by coordinate maps, such as (x, y) maps to $(x + 3, y)$ for a translation.

Discussion: This standard is fully met.

5. *Does the curriculum enable all students to use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides?*

Score: 3

Evidence:

- *(6) Shapes and Designs.* On p. 60, problems 12-16, students figure out what four coordinates to plot to make a rectangle, and a parallelogram that is not a rectangle. They also plot triangles.
- *(7) Stretching and Shrinking.* In IV6, Turtle Math is used to draw rectangles, equilateral triangles and right trapezoids. The figures are scaled with Scale Tool and observed on a coordinate grid. Slopes are not yet used to analyze parallel or perpendicular lines.
- *(7) Moving Straight Ahead.* On p. 50, two lines are given on a coordinate grid and students are to conclude whether they are perpendicular based on their slopes. No geometric shapes are examined here.
- *(8) Looking for Pythagoras.* On p. 68, students find slopes of sides of figures from dot paper and their coordinates. They see how the slopes are related if the lines are perpendicular. In problem 15a, they look at the slopes of lines in a triangle formed by the diameter and chords of a circle.

Discussion: This standard is fully met.

6. *Does the curriculum enable all students to describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling?*

   Score: 3

   Evidence:

   - *(6) Shapes and Designs.* On pp. 52-54, flips and turns are introduced. No translations or scaling are done yet.
   - *(6) Covering and Surrounding.* On p. 67, problem 23b., students discover that flipping a triangle over can make a parallelogram when combined with the original triangle. They use this fact to deduce the area of a general triangle from the area of a parallelogram.
   - *(8) Stretching and Shrinking.* In IV3, students build similar figures called Wumps using scaling. Their positions are plotted on a cartesian grid and the transformations between the similar figures are determined. In IV4, students build larger similar figures, such as triangles, from smaller similar ones. The small triangle in the larger one is referred to as a rep-tile.
   - *(8) Kaleidoscopes, Hubcaps, and Mirrors.* This entire book is on symmetry transformations of flips, turns, and slides that lead to congruent figures. The students use these transformations and select tools to do these operations to make tesselations.

   Discussion: This standard is fully met.

7. *Does the curriculum enable all students to examine the congruence, similarity, and line of rotational symmetry of objects using transformations?*

   Score: 3

   Evidence:

   - *(6) Shapes and Designs.* On p. 5, simple rotational symmetry is introduced.
   - *(7) Stretching and Shrinking.* In IV2, coordinates of a Wump are plotted. These coordinates are scaled and a new Wump is plotted. The similarity is investigated. On p. 27d, the teacher is told the rule that (x, y) maps to (nx, ny) is what produced the figures.
   - *(8) Kaleidoscopes, Hubcaps, and Mirrors.* Congruency is analyzed under similarity transformations. Tesselations are produced. Students locate lines of rotational symmetry.

   Discussion: This standard is fully met.

8. *Does the curriculum enable all students to draw geometric objects with specified properties, such as side lengths or angle measures?*

   Score: 2

   Evidence:

   - *(6) How Likely Is It?* Students make spinners for representing unequal probabilities. This requires angle measurements.
   - *(6) Covering and Surrounding.* On p. 49, students construct parallelograms with specified measures. In problem 5.2, students use centimeter grid paper to draw rectangles and parallelograms with given dimensions and areas.
   - *(7) Stretching and Shrinking.* On pp. 5–7, students use a rubber-band stretcher to draw similar figures that scale to twice or three times the original size.
   - *(8) Kaleidoscopes, Hubcaps, and Mirrors.* In the Unit Project, described on p. 71, students use dot paper, an angle ruler, and a protractor to make symmetry transformations and tesselations.

   Discussion: This standard is adequately met. Students draw some simple objects with specified lengths and areas in *Covering and Surrounding.*

9. *Does the curriculum enable all students to use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume?*

   Score: 3

   Evidence:

   - *(6) Ruins of Montarek.* This unit is entirely about spatial visualization. Students read information from a drawing to reason about and observe 3D objects to make 2D drawings. Students build from plans. For cubical objects, they determine the number of small cubes (volume) needed from a 2D representation. They learn to figure out how much surface area a 3D object has when part of it is hidden from view.
   - *(7) Filling and Wrapping.* In IV1, a net for a unit cube is given on p. 5, a net for a rectangular box on p. 7, and a net for a cylinder on p. 39. Students make these nets and calculate the surface area and the volume of the 3D figures produced.

   Discussion: This standard is fully met.

10. *Does the curriculum enable all students to use visual tools such as networks to represent and solve problems?*

    Score: 3

    Evidence:

- *(6) Shapes and Designs.* On p. 57, the shortest path through a grid is found. Students explain why it is the shortest.
- *(8) Clever Counting.* IV3, students use networks to analyze the number of paths between cities and work other counting problems. Students create networks that satisfy given constraints.

Discussion: This standard is fully met.

11. *Does the curriculum enable all students to use geometric models to represent and explain numerical and algebraic relationships?*

Score: 3

Evidence:

- *(7) Filling and Wrapping.* In IV4, students examine two-dimensional nets for cylinders. They answer questions about the surface area of a cylinder from the geometric models.
- *(8) Looking for Pythagoras.* On p. 40g, the teacher is told how to use a geometric model to see that $(a+b)^2 = a^2+2ab+b^2$ and to prove the Pythagorean Theorem.
- *(8) Frogs, Fleas, and Painted Cubes.* Rectangles are used to go from story problems to algebraic expressions for quadratic relationships.
- *(8) Say It With Symbols.* Areas of rectangles are used to explain the distributive property of multiplication over addition on p. 1c and p. 20.

Discussion: This standard is fully met.

12. *Does the curriculum enable all students to recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life?*

Score: 3

Evidence:

- *(6) Shapes and Designs.* In IV1, tesselations with hexagons that are related to honeycombs are shown. On p. 4, a picture of the Pentagon is shown.
- *(6) Covering and Surrounding.* The ideas of perimeter and area are used in the context of floor plans. The Unit Project is to design a park that meets various area constraints.
- *(6) Ruins of Montarek.* Nets are shown for surface views. Blueprints are shown.
- *(7) Stretching and Shrinking.* On p. 61, the mirror method is used with similar triangles to find the heights of buildings and the width of irregular shapes. On p. 71, similar triangles are used to calculate distances in astronomy.

- *(7) Filling and Wrapping.* In IV7, volumes of irregular 3D objects are found by placing them in water. This is a connection to physics.
- *(8) Looking for Phytagoras.* In IV4, the Pythagorean Theorem is used to compute side lengths in applications, such as finding out if a barn wall is really perpendicular to the ground and how far a catcher must throw the ball to the 2nd baseman.
- *(8) Kaleidoscopes, Hubcaps, and Mirrors.* Symmetry transformations are used to create interesting artistic tesselations.

Discussion: This standard is fully met.

B5.2 Geometry Summary

Question	CMP
Geometry 1.	3
Geometry 2.	3
Geometry 3.	3
Geometry 4.	3
Geometry 5.	3
Geometry 6.	3
Geometry 7.	3
Geometry 8.	2
Geometry 9.	3
Geometry 10.	3
Geometry 11.	3
Geometry 12.	3

Table 3: Summary of NCTM Geometry Standard Results

The scores in the table above show that the curriculum fully meets eleven of the twelve geometry standards and adequately meets one standard. The lower score was given because the construction of objects to specification was only done for very simple objects, such as parallelograms and rectangles. The evidence points to other issues worth mentioning that the scoring did not reflect.

- We did not find evidence that students create or critique inductive or deductive arguments in 6th grade.
- Geometric models are not used to represent and explain numerical and algebraic relationships until 8th grade.

B6 Measurement Standard

B6.1 Measurement Questions and Evidence

Note: In what follows IV1 refers to Investigation 1, IV2 to Investigation 2, etc.

1. *Does the curriculum enable all students to understand both metric and customary systems?*

   Score: 3

   Evidence:

   - *(6) Bits and Pieces I.* An enlarged version of a 1 cm unit is divided into 10 parts on p. 60.
   - *(6) Covering and Surrounding.* An area is given in m^2 units.
   - *(7) Data About Us.* On p. 15, conversions between the systems are given, e.g. 1 $inch = 2.54$ cm. Students select a familar object whose measurements are known in the customary system and convert it to the metric system to get a "benchmark" idea of its size in metric units. Grams and kilograms are given as units of mass. Ounces and pounds are given as units of weight. In the Summarize section on p. 22c, the teacher attaches the proper units to length, area, weight or mass, temperature, and time.
   - *(8) Looking for Pythagoras.* Students use a centimeter ruler to estimate the length of a side of a square.

   Discussion: This standard is fully met.

2. *Does the curriculum enable all students to understand relationships among units and convert from one unit to another within the same system?*

   Score: 1

   Evidence:

   - *(7) Filling and Wrapping.* In IV7, the cm^3 and millileter relationship is given.
   - *(8) Clever Counting.* On p. 12a, the conversion of time units is seen. No conversion of units of length within the same system is found.
   - *(8) Frogs, Fleas, and Painted Cubes.* On p. 154, 3 $ft. = 1$ $yd.$ is found in an Additional Practice problem.

   Discussion: This standard is not adequately met. Students convert between systems, but hardly at all within the same system.

3. *Does the curriculum enable all students to understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume?*

   Score: 3

   Evidence:

   - *(6) Shapes and Designs.* Students measure angles in degrees with an angle ruler.
   - *(6) Covering and Surrounding.* Throughout this unit, students use centimeter-square grid paper to measure or approximate the area of figures. These figures range from simple qualrilaterals, for which they find the exact areas, to complex two-dimensional shapes, such as hands and feet.
   - *(7) Data Around Us.* In IV2, students select a unit of appropriate size to describe data. These units are for length, area, surface, and volume in the context of the Exxon Valdez oil spill. (They must select units appropriate for very large numbers.)
   - *(8) Looking for Pythagoras.* In IV2, p. 19, students find areas of two-dimensional shapes that are drawn on dot paper. The unit of length is taken to be the distance between two dots.

   Discussion: This standard is fully met.

4. *Does the curriculum enable all students to use common benchmarks to select appropriate methods for estimating measurements?*

   Score: 3

   Evidence:

   - *(6) Shapes and Designs.* In IV3, students estimate angles using common angular turns, such as 45^o and 90^o, as benchmarks.
   - *(7) Data Around Us.* In IV2, students invent their own benchmarks to rewrite the Exxon Valdez article in units more understandable to their classmates.
   - *(8) Looking for Pythagoras.* In IV5, students locate irrational numbers close to a benchmark fraction.

   Discussion: This standard is fully met.

5. *Does the curriculum enable all students to select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision?*

   Score: 3

   Evidence:

- *(6) Bits and Pieces I.* Students use paper strips for fractions to measure the thermometer for their fund-raising event. On p. 21, they use fraction strips to explore the equivalence of fractions. In IV3, students have to determine how to cut a pan of brownies into equal pieces. The teacher is told that it is hard for students to make reasonable representations for doing this – that some students will select rulers and others will struggle to find a way to do it.
- *(6) Data About Us.* On p. 52b, it is suggested that students use a measuring tape to measure heights for a statistics project.
- *(6) Shapes and Designs.* Students use an angle ruler to measure angles approximately.
- *(6) Covering and Surrounding.* On p. 67, rules of thumb are used to think about the sizes of measurements.
- *(7) Filling and Wrapping.* In IV7, students study the relationship between a cubic centimeter and a milliliter. Then they use the water displacement method to find the volume of irregular objects to a certain precision.
- *(8) Looking for Pythagoras.* In IV2, p. 19, students use a centimeter ruler to measure the side length of a square (area of two square units) that was drawn on centimeter grid paper. Then a calculator is used to find the square root of two and precision of the measurement discussed.

Discussion: This standard is fully met.

6. *Does the curriculum enable all students to develop and use formulas to determine the circumference of circles and areas of triangles, parallelograms, trapezoids, circles, and develop strategies to find areas of more complex shapes?*

Score: 3

Evidence:

- *(6) Covering and Surrounding.* In IV5, students develop formulas for finding areas of triangles, rectangles, parallelograms, and trapezoids. The area of a circle is found approximately.
- *(7) Variables and Patterns.* On p. 49, the circumference of a circle is written symbolically as $C = \pi d$.
- *(8) Looking for Pythagoras.* In IV2, students find areas of figures on dot paper. They also use the strategy of dividing the figure into commonly known figures or by finding a length that is the side of a square of known area.

Discussion: This standard is fully met.

7. *Does the curriculum enable all students to develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders?*

   Score: 3

   Evidence:

   - *(7) Filling and Wrapping.* In IV2, the surface area of a rectangular prism is investigated. In IV3, the volumes of boxes and prisms are studied. In IV4, the surface area and volumes of cylinders are studied. In IV5, the volume of a cone is studied. On p. 55, problem 12b, students must describe methods for finding the surface area and the volume of a pyramid.

   Discussion: This standard is fully met.

8. *Does the curriculum enable all students to solve problems involving scale factors, using ratio and proportion?*

   Score: 3

   Evidence:

   - *(6) Covering and Surrounding.* Students use scale to plan the layout of a park.
   - *(7) Stretching and Shrinking.* On p. 61, mirrors are used to find heights using similar triangles.
   - *(7) Filling and Wrapping.* In IV6, a scale factor is calculated to make a similar rectangular prism with specified volume.
   - *(7) Comparing and Scaling.* On p. 41, problem 4.2b, students use a ratio table to solve problems involving proportion. On p. 95 in the Question Bank, the scale of 1 *in.* = 1 *million mi.* is used for a solar system calculation.

   Discussion: This standard is fully met.

9. *Does the curriculum enable all students to solve simple problems involving rates and derived measurements for such attributes as velocity and density?*

   Score: 3

   Evidence:

   - *(7) Variables and Patterns.* In IV4, rate-time-distance relationships are studied using tables, graphs, and equations. Population density (number of people per area) is also studied.
   - *(7) Comparing and Scaling.* On p. 42, problem 4.3, average speed is calculated for a problem involving speeding up and slowing down. It is made clear that a single linear equation cannot describe the situation, that it is piecewise linear.

   Discussion: This standard is fully met. (We note that density in the sense of mass per unit volume is not included – only population density is studied.)

B6.2 Measurement Summary

Question	Singapore
Measurement 1.	3
Measurement 2.	1
Measurement 3.	3
Measurement 4.	3
Measurement 5.	3
Measurement 6.	3
Measurement 7.	3
Measurement 8.	3
Measurement 9.	3

Table 4: Summary of NCTM Measurement Standard Results

The scores in the table above show that the curriculum fully meets eight of the nine measurement standards, and does not adequately meet one standard. The lower score was given because students work minimally with measurement conversions within the same measurement system. The evidence points to another issue worth mentioning that the scoring did not reflect.

- In the problems students do, density always refers to population density (which has units of mass per unit area). No examples from physics were included where density is interpreted as mass per unit volume.

B7 Data and Probability Standard

B7.1 Data and Probability Questions and Evidence

Note: In what follows IV1 refers to Investigation 1, IV2 to Investigation 2, etc.

1. *Does the curriculum enable all students to formulate questions, design studies and collect data about a characteristic shared by two populations or different characteristics within one population?*

   Score: 3

   Evidence:

   - *(6) Data About Us.* On p. 42 students study two characteristics (arm span and height) of a population. They use scatter plots to see how these characteristics are related. The Unit Project is to design a study and collect data about the question, "Is Anyone Typical?"
   - *(8) Samples and Populations.* In IV2, students design and conduct surveys. In IV3, p. 40, they learn how to take random samples and how sample size affects accuracy of the results.

   Discussion: This standard is fully met.

2. *Does the curriculum enable all students to select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots?*

   Score: 3

   Evidence:

   - *(6) Data About Us.* On p. 8, histograms and frequency plots are used. On p. 42, scatter plots are used. In the ACE problems on p. 49, students select and create these representations.
   - *(8) Samples and Populations.* On p. 7 box and whisker plots are used, and on p. 13 scatter plots are used to present results.

   Discussion: This standard is fully met.

3. *Does the curriculum enable all students to find, use, and interpret measures of center and spread, including mean and interquartile range?*

   Score: 3

   Evidence:

   - *(6) Data About Us.* The median is introduced on p. 12 and the mode on p. 9. Throughout IV5, students get a comprehensive view of mean by comparing different looking graphs that all represent samples with the *same* mean. The range of a data set is also discussed.

- *(8) Samples and Populations.* On p. 9, the interquartile range is discussed in relation to box and whisker plots.

Discussion: This standard is fully met.

4. *Does the curriculum enable all students to discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots?*

   Score: 3

   Evidence:

   - *(6) Data About Us.* Histograms are on p. 8, stem-and-leaf plots on p. 32, and scatter plots on p. 42. In IV1, students reflect on how tables and line plots are alike and how they are different. In prob 8a, p. 39, students have to describe which type of representation is best to use with the data.
   - *(8) Samples and Populations.* On p. 23, in the Reflection, students describe in what situations various representations are useful.

   Discussion: This standard is fully met.

5. *Does the curriculum enable all students to use observations about differences between two or more samples to make conjectures about the populations from which samples were taken?*

   Score: 3

   Evidence:

   - *(7) Comparing and Scaling.* In IV5, students use the capture-tag-recapture technique to draw samples from a population to estimate the size of the population. They do this by tagging 100 beans from a large container of beans. Applications to the size of a deer population are discussed.
   - *(8) Samples and Populations.* In IV2 and IV4, students use samples to make predictions about a population and study, on p. 41, how sample size affects results.

   Discussion: This standard is fully met.

6. *Does the curriculum enable all students to make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit?*

   Score: 3

   Evidence:

- *(6) Data About Us.* In IV4, the relationship of arm span to height is studied from scatter plots. A line $y = x$ is drawn through the data and students make statements about data that lie above or below the line.
- *(8) Thinking with Mathematical Models.* On p. 7, lines of fit to the data are discussed. Students draw a line that looks as if it fits the data and then they find its equation.

Discussion: This standard is fully met.

7. *Does the curriculum enable all students to use conjectures to formulate new questions and plan new studies to answer them?*

   Score: 0

   Evidence:

   - *(6) Data About Us.* Students plan a unit long study. On p. 25, the question "Can the question be answered by the graph? If so, what is the answer. If not, what additional information would be needed?" is a start toward designing new studies, but it is not evident from the curriculum that they actually do the new design.
   - *(6) How Likely Is It?* On p. 49, problem 6.1, students create and do a simulation to answer probability questions.
   - *(8) Samples and Populations.* On p. 53, students explain what is wrong with the conjectures made by others. On p. 63, they design a study and carry it out as a Unit Project. They make conjectures to know what questions to include. It is not clear whether based on the results they obtain, that they plan a second study.

   Discussion: This standard is not met. The curricular materials do not explicitly ask students to design further studies based on results found from an initial investigation.

8. *Does the curriculum enable all students to understand and use appropriate terminology to describe complementary and mutually exclusive events?*

   Score: 2

   Evidence:

   - *(6) How Likely Is It?* Students certainly see complementary events, such as Red or Not Red. The word complementary is not used.
   - *(7) What Do You Expect?* Mutually exclusive events are in this unit, but these words are not used.

   Discussion: This standard is adequately met. The desired vocabulary is not used, nor is it in the glossary of any of the probability units. Students do however work with mutually exclusive and complementary events.

9. *Does the curriculum enable all students to use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations?*

   Score: 3

   Evidence:

   - *(6) How Likely Is It?* Students do simulations and use probability.
   - *(7) What Do You Expect?* In IV5, students use an example from basketball to make conjectures about expected values. They predict the number of baskets made in a one-for-one free throw situation with 100 trips to the free throw line. This requires the principle of proportionality. The Unit Project requires students to design a carnival game, calculate the expected payout (make the game so the school wins money!), calculate the probability of winning, simulate it, and convince the carnival committee (in writing) to accept it.

   Discussion: This standard is fully met.

10. *Does the curriculum enable all students to compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models?*

    Score: 3

    Evidence:

    - *(7) What Do You Expect?* In IV4, the area model, lists, and probability trees are used to solve two-stage games, and maze problems with expected values that involve dependent events.

    Discussion: This standard is fully met.

B7.2 Data and Probability Summary

Question	CMP
Data Anal. and Prob. 1.	3
Data Anal. and Prob. 2.	3
Data Anal. and Prob. 3.	3
Data Anal. and Prob. 4.	3
Data Anal. and Prob. 5.	3
Data Anal. and Prob. 6.	3
Data Anal. and Prob. 7.	0
Data Anal. and Prob. 8.	2
Data Anal. and Prob. 9.	3
Data Anal. and Prob. 10.	3

Table 5: Summary of NCTM Data Anal. and Prob. Standard Results

The scores in the table above show that the curriculum fully meets eight of the ten standards, adequately meets one standard, and does not meet one standard. The lower scores were given because students do not design further studies after an initial study has been completed and do not use the vocabulary of complementary and mutually exclusive events. The evidence points to another issue worth mentioning that the scoring did not reflect:

- With the exception of the capture-tag-recapture technique in the 7th grade *Comparing and Scaling* unit, there is no statistics in 7th grade CMP.

B8 Problem Solving Standard

B8.1 Problem Solving Questions and Evidence

Note: In what follows IV1 refers to Investigation 1, IV2 to Investigation 2, etc.

1. *Does the curriculum enable all students to build new math knowledge through problem solving?*

   Score: 3

   - *(6) Prime Time.* On p. 48, students build the Prime Factorization Theorem by finding the longest string of factors of a number.
   - *(6) Bits and Pieces I.* Students build knowledge of the equivalence between fractions, decimals, and percents by working with a 100 grid representation. On p. 23, they build knowledge of equivalent fractions by constructing fraction strips and matching them up on a number line.
   - *(6) Shapes and Designs.* On p. 16, students build knowledge of the triangle inequality by trying to make a triangle from any three given lengths (which is not always possible).
   - *(6) Covering and Surrounding.* In problem 19a, p. 65, students build their own formula for the area of a trapezoid by construction and reasoning.
   - *(6) Bits and Pieces II.* By working in groups, students come up with the algorithms for addition and subtraction of fractions on p. 48. They discover the standard algorithm for multiplication of fractions on p. 68.
   - *(7) Filling and Wrapping.* In IV4, students build a cylinder from a two-dimensional net made on centimeter grid paper. They build up the formula for its surface area and volume.
   - *(8) Thinking With Mathematical Models.* Students build new knowledge about the nature of differences in linear and nonlinear relationships by using paper models of bridges that demonstrate each case.

   Discussion: This standard is fully met.

2. *Does the curriculum enable all students to solve problems that arise in math and in other contexts?*

   Score: 3

   Evidence:

   - *(6) Prime Time.* On p. 38, students solve problems involving cycles of locusts using common multiples.

- *(6) Data About Us.* On p. 84, social contexts of political campaigns are used to discuss interpretations of average.
- *(6) Ruins of Montarek.* Architectural plans and side views are used for spatial visualization.
- *(6) Bits and Pieces II.* Sales taxes, tips, discounts, and ordering from a catalog are used as contexts to study percents.
- *(7) Stretching and Shrinking.* On p. 71-72, astronomy problems of planetary distances are used to motivate study with similar triangles.
- *(7) Comparing and Scaling.* Advertising and supermarket contexts are used to study rates and ratios.
- *(7) Accentuate the Negative.* Temperatures below zero are used to relate to negative numbers.
- *(8) Growing, Growing, Growing.* The half life of Iodine-124 is simulated to study exponential decay.
- *(8) Clever Counting.* In IV2 counting all the possibilities is motivated by finding how many lock combinations a thief would have to try if there are n numbers on the lock and r numbers make one combination.

Discussion: This standard is fully met.

3. *Does the curriculum enable all students to apply and adapt a variety of appropriate strategies to solve problems?*

   Score: 3

   Evidence:

   - *(6) Prime Time.* On p. 16c the teacher is told to give the students a chance to suggest a strategy to solve the problem.
   - *(6) Data About Us.* On pp. 15–21, students adapt the concept of a bar graph to make a double-bar graph and a stem-and-leaf plot to make a double one to better present results from two populations.
   - *(7) Filling and Wrapping.* In IV1, students investigate volumes of objects by filling them with unit cubes. In IV7, they use the strategy of seeing how much water the object displaces to find an estimate of its volume.
   - *(8) Looking for Pythagoras.* In IV2, areas of complicated 2D shapes are found by splitting them into shapes of known areas and applying known strategies.
   - *(8) Samples and Populations.* On p. 28, five sampling strategies are given. Students must analyze the sampling plans of four other groups and tell the pros and cons.

   Discussion: This standard is fully met.

4. *Does the curriculum enable all students to monitor and reflect on the process of mathematical problem solving?*

   Score: 3

   Evidence:

   - *(6 to 8) All Units.* CMP makes it explicit that students should monitor and reflect on the process of mathematical problem solving. At the end of the investigation (usually 4 or 5) in each unit, students answer reflection questions. These questions require writing, talking to other students, and talking to the teacher.
   - *(6) Bits and Pieces II.* In the IV4 Reflection, students write about how to add and subtract fractions with unlike and like denominators and how to multiply fractions. On p. 19, it states "One of the powerful things about math is that you can often find a way to solve one problem that will also work for solving similar problems." Five problems are then given and students have to find one strategy that works for all of them and compare with each other's strategy.
   - *(7) Comparing and Scaling.* In IV2, students write about when it is good to use percent in rate comparisons.

   Discussion: This standard is fully met.

B8.2 Problem Solving Summary

Question	CMP
Problem Solving 1.	3
Problem Solving 2.	3
Problem Solving 3.	3
Problem Solving 4.	3

Table 6: Summary of NCTM Problem Solving Standard Results

The scores in the table above show that the curriculum fully meets all four of the standards.

B9 Reasoning and Proof Standard

B9.1 Reasoning and Proof Questions and Evidence

Note: In what follows IV1 refers to Investigation 1, IV2 to Investigation 2, etc.

1. *Does the curriculum enable all students to recognize reasoning and proof as fundamental aspects of math?*

   Score: 3

   Evidence:

   - *All units.* Thoughout CMP students are asked to explain their reasoning. They answer why and why not and explain type questions in ACE problems and orally during the Investigations.
   - *(6) Prime Time.* On p. 13, students are asked, "What is the best first move on a 49-board? Why?"
   - *(6) How Likely Is It?* In problem 5.1, p. 43, students reason what a good strategy would be to win a probability game like Roller Derby.
   - *(6) Covering and Surrounding.* On p. 33, the statement, "Find all possible pentominos and say how you know you have them all," illustrates the notion of proof. In IV4, students add tiles to pentominos and reason about how the area and perimeter changes with each addition.
   - *(7) Bits and Pieces II.* On pp. 53c–53d, the curriculum promotes reasoning in the classroom by giving the teacher a dialogue used by students. The dialogue shows how students reason about the changing distribution of land based on clues they are given in one of the investigations.
   - *(8) Looking for Pythagoras.* On p. 29, the definition of a theorem is given.
   - *(8) Clever Counting.* This entire unit is about using combinatorial arguments to make math conjectures about who the mystery person was based on clues given.

   Discussion: This standard is fully met.

2. *Does the curriculum enable all students to make and investigate math conjectures?*

   Score: 3

   Evidence:

   - *(6) Prime Time.* The word conjecture is defined on p. 28. Students make a conjecture and build a model to justify it on p. 29. For example, "The sum of two even numbers is an odd number."

- *(6) Shapes and Designs.* Students conjecture about the sum of interior angles of regular polygons on p. 44.
- *(7) Stretching and Shrinking.* In problems 2–5 on p. 23, students predict how changes in the rule affect changes in a figure. In problems 2b and 2c on p. 8, they predict what would happen as the anchor point moves up and down. They test this prediction and choose a new anchor point to see if their prediction is still true.
- *(8) Looking for Pythagoras.* In IV2, after experimentation, students make a conjecture about the relationship of the sides of a right triangle. This leads students to prove the Pythagorean Theorem geometrically.
- *(8) Clever Counting.* This entire unit is about using combinatorial arguments to make math conjectures about who the mystery person is based on clues given.
- *(8) Hubcaps and Kaleidoscopes.* In problem 27, p. 57, students are asked, "Investigate what happens when you rotate a figure 180 degrees about a point and then rotate the image 180 degrees about a different point. Is the combination of the two rotations equivalent to a single transformation?" Students are told to test several cases, and make a conjecture about the result.

Discussion: The standard is fully met.

3. *Does the curriculum enable all students to develop and evaluate math arguments and proofs?*

Score: 3

Evidence:

- *(6) Prime Time.* On p. 28, students use models to justify or prove their math conjectures.
- *(6) How Likely Is It?* Students are asked does the statement, "Nine out of ten dentists surveyed recommend sugarless gum for their patients that chew gum," mean that 90% of dentists think patients should chew sugarless gum.
- *(7) Comparing and Scaling.* In problem 1.2 on p. 7, students evaluate ratio and proportion statements for correctness.
- *(7) Stretching and Shrinking.* On p. 54, students determine whether triangles are similar and if so give a scale factor based on the lengths of the sides. In the Reflection, on p. 40, students are asked, "How can you decide whether two figures are similar?" On p. 58, students develop an argument for whether two rectangles are similar or not similar.
- *(7) Filling and Wrapping.* In IV3, students are discovering an inductive argument for the volume of any right prism. In problem 18, p. 35, students explain how to find the volume of a right prism with an irregularly shaped base.

- *(8) Clever Counting.* This entire unit is about using combinatorial arguments to make math conjectures about who the mystery person is based on clues given.
- *(8) Samples and Populations.* On p. 53, students figure out what is wrong with another student's logic.
- *(8) Say It With Symbols.* On p. 40, students develop a math argument for why three different strategies would work for finding the area of a trapezoid.
- *(8) Looking for Pythagoras.* In IV2, students figure out how some given puzzle pieces (that illustrate the Pythagorean Theorem) fit together. In IV3, p. 27, the Pythagorean Theorem is introduced. Students make a table with the lengths of the sides and their squares, and use this to conjecture about their relationship based on the pattern seen. On p. 29, a definition of *theorem* is given. Students use geometric ideas to prove this theorem. On p. 40g, an algebraic proof based on a geometrical proof is given to the teacher.
- *(8) Hubcaps and Kaleidoscopes.* On p. 49, problem 2b, students are asked to find a transformation that moves one circle to another to check for congruence. On p. 51, problem 5 asks the same question for congruence of triangles. In problem 27, p. 57, students are asked, "Investigate what happens when you rotate a figure 180 degrees about a point and then rotate the image 180 degrees about a different point. Is the combiniation of the two rotations equivalent to a single transformation?" Test several cases, and make a conjecture about the result. This is an example of inductive reasoning.

Discussion: This standard is fully met.

4. *Does the curriculum enable all students to select and use various types of reasoning and methods of proof?*

Score: 3

Evidence:

- *(6) Prime Time.* On p. 12, students are asked "How would you check to see if one number is a factor of another?" On p. 28, students use models to justify or prove their math conjectures.
- *(6) Shapes and Designs.* Discovery and reasoning are coupled to show that not every three side lengths necessarily make a triangle.
- *(7) Stretching and Shrinking.* In problem 4.2, p. 43, students must select a way to scale a picture to a given size or say why it can't be done in a such a way that the new picture is similar to the original. On p. 54, students determine whether triangles are similar and if so give a scale factor based on the lengths of the sides. In the Reflection, on p. 40, students are asked, "How can you decide whether two figures are similar?"

- *(7) Comparing and Scaling.* On p. 11, students choose a way to report results of an experiment that uses a spinner. The ways suggested include ratios, percents, or differences. On p. 13, students explain which reasoning methods should be used in particular circumstances.
- *(7) Filling and Wrapping.* In the Reflection on p. 72, students are asked to prove the relationship between cubic centimeters and milliliters. This leads to the water displacement method. In IV3, students are discovering an inductive argument for the volume of any right prism. In problem 18, p. 35, students explain how to find the volume of a right prism with an irregularly shaped base.
- *(8) Thinking with Math Models.* Students use a graph to reason about a linear relationship. As x increases, what happens to y. Does this change when the slope is negative?
- *(8) Looking for Pythagoras.* Students have to select a way to draw a line through dot paper that does not intersect other dots. In IV3, p. 27, the Pythagorean Theorem is introduced. Students make a table with the lengths of the sides and their squares, and use this to conjecture about their relationship based on the pattern seen. On p. 29, a definition of *theorem* is given. Students use geometric ideas to prove this theorem. On p. 40g, an algebraic proof based on a geometrical proof is given to the teacher.
- *(8) Clever Counting.* On p. 46d, reasoning is given to the teacher that leads to formulas for permutations and combinations. On p. 18, problem 2.3, students use inductive reasoning to find an equation for the number of combinations, c, in terms of the number of marks, m, on a lock.
- *(8) Frogs, Fleas, and Painted Cubes.* On p. 51c, the teacher pages explain that one can often first prove what something is not. For example, prove that the tables of values can not possibly represent a linear relationship.
- *(8) Hubcaps and Kaleidoscopes.* In problem 27, p. 57, students are asked, "Investigate what happens when you rotate a figure 180 degrees about a point and then rotate the image 180 degrees about a different point. Is the combination of the two rotations equivalent to a single transformation?" Students are asked to test several cases, and make a conjecture about the result. This is an example of inductive reasoning. On p. 49, problem 2b, students are asked to find a transformation that moves one circle to another to check for congruence. On p. 51, problem 5 asks the same question for congruence of triangles. Students prove congruence by the similarity transformations of reflection, rotation, and translation.

Discussion: This standard is fully met.

B9.2 Reasoning and Proof Summary

Question	CMP
Reason and Proof 1.	3
Reason and Proof 2.	3
Reason and Proof 3.	3
Reason and Proof 4.	3

Table 7: Summary of NCTM Reason and Proof Standard Results

The scores in the table above show that the curriculum fully meets all four of the standards.

B10 Communication Standard

B10.1 Communication Questions and Evidence

Note: In what follows IV1 refers to Investigation 1, IV2 to Investigation 2, etc.

1. *Does the curriculum enable all students to organize and consolidate their math thinking through both written and oral communication?*

   Score: 3

   Evidence:

   - *(6th to 8th) All Units.* After each investigation in each unit, the Reflection section asks students to write about and discuss their mathematical thinking. All units have group work. Most units have a Unit Project which requires both written and oral communication.
   - *(6) Bits and Pieces I.* Students are asked to explain if two classes raised the same amount of money if they both reached 3/5 of their respective goals. (The classes had different goals.)
   - *(6) Ruins of Montarek.* During the Unit Project, students must communicate in order to make a model building. They have to submit a written report as well.
   - *(6) Bits and Pieces II.* On p. 63 in the Reflection, students are to think about what pattern there is when multiplying one fraction by another. This could lead to the standard algorithm of multiplying numerators and denominators. This will be shared with the class during the Summarize part of the lesson.

   Discussion: This standard is fully met.

2. *Does the curriculum enable all students to communicate math thinking coherently and clearly to peers, teachers, and others? (both oral and written)*

   Score: 3

   Evidence:

   - *(6th to 8th) All Units.* After each investigation in each unit, the Reflection section asks students to write about and discuss their mathematical thinking with other students and the teacher. All units have group work and most have a Unit Project.
   - *(6) Prime Time.* Students make up word problems that involve common factors and common multiples.
   - *(6) Bits and Pieces I.* In the Reflection, students are asked to explain to others and to write in their journals how to tell which is bigger .57 or .559.

- *(7) What Do You Expect?.* In the Unit Project, students design, build, simulate, and analyze a new carnival game. They write a persuasive report to the carnival committee to accept the game at the next carnival.

Discussion: This standard is fully met.

3. *Does the curriculum enable all students to analyze and evaluate math thinking and strategies of others?*

   Score: 3

   Evidence:

   - *(6th to 8th) All Units.* After each investigation in each unit, the Reflection section asks students to write about and discuss their mathematical thinking with other students and the teacher. All units have group work and most have a Unit Project.
   - *(6) Data About Us.* Students analyze statements made by political candidates and decide which are correct.
   - *(6) Bits and Pieces I.* On p. 52b, the Teacher's Guide contains a dialogue where two students communicate to analyze each other's thinking.
   - *(6) Bits and Pieces II.* In problem 3c, p. 8, students evaluate whether the tip was added before or after the tax was added to the bill.
   - *(7) Stretching and Shrinking.* In problem 11, p. 51, students are asked to provide the rule that Samantha used to make the new triangle.
   - *(8) Looking for Pythagoras.* Students are given pieces that make up a puzzle that can be used to prove the Pythagorean Theorem. They have to decide how the theorem can be proven using these pieces.
   - *(8) Say It With Symobls.* On p. 40, three strategies for finding the area of a trapezoid are given and students must explain why each works.

   Discussion: This standard is fully met.

4. *Does the curriculum enable all students to use the language of math to express math ideas precisely?*

   Score: 3

   Evidence:

   - *(6) Prime Time.* On p. 15, "Is there a largest perfect number?" gives a glimpse of questions a mathematician might ask.
   - *(6) Shapes and Designs.* On p. 21c, the teacher is told how a mathematician would write and discuss the triangle inequality that could be shared with the students.

- *(7) Comparing and Scaling.* On p. 51e, the teacher is told that the changing speed problems are related to *piecewise linear graphs.* In problem 4.4, p. 43, students write an equation that relates the cost C and the number of beads x for each type of bead.
- *(8) Looking for Pythagoras.* On p. 29, the definition of a theorem is given. On p. 1g, a proof by contradiction is given to the teacher that the square root of 2 is irrational. On p. 50, the notion of generalization is expressed by asking whether the Pythagorean idea works for more general 2D areas attached to the sides of a right triangle.
- *(8) Hubcaps and Kaleidoscopes.* In problem 27, p. 57, students are asked, "Investigate what happens when you rotate a figure 180 degrees about a point and then rotate the image 180 degrees about a different point. Is the combination of the two rotations equivalent to a single transformation?" Students are asked to test several cases, and make a conjecture about the result. This is an example of using correct mathematical terminology in a student problem. On p. 59, the term symmetry is precisely defined: "A geometric figure is symmetric if a reflection or rotation of the figure produces an image that matches the original figure exactly." On p. 63, the inverse of a symmetry transformation is defined. On p. 58j (to the teacher), equality is compared to congruence. The teacher is told if the students are ready, such a discussion could be done informally with them.

Discussion: This standard is fully met.

B10.2 Communication Summary

Question	CMP
Communication 1.	3
Communication 2.	3
Communication 3.	3
Communication 4.	3

Table 8: Summary of NCTM Communication Standard Results

The scores in the table above show that the curriculum fully meets all four of the standards.

B11 Connection Standard

B11.1 Connection Questions and Evidence

Note: In what follows IV1 refers to Investigation 1, IV2 to Investigation 2, etc.

1. *Does the curriculum enable all students to recognize and use connections among math ideas?*

   Score: 3

   Evidence:

   - *(6 to 8) All Units.* After each investigation, students do "C" or Connection problems that require connections to mathematics previously studied.
   - *(6) Prime Time.* Students use areas of rectangles and sides of rectangles to find all the factors of whole numbers.
   - *(6) Shapes and Designs.* On p. 41i for the teacher, a suggestion is made to get the students to make a line plot of all the angle measurements students found when measuring a a particular angle to see the spread and mean. This is a connection to statistics. Angles are also connected to the positioning of the hands of a clock.
   - *(6) How Likely Is It?* Students read bar graphs to see if events are equally likely. On p. 45, the event that the sum showing on two dice will be prime is a connection to *Prime Time.*
   - *(8) Frogs, Fleas, and Painted Cubes.* Students recognize that length is linear in its units and area is quadratic.

   Discussion: This standard is fully met.

2. *Does the curriculum enable all students to understand how math ideas interconnect and build to create a coherent whole?*

   Score: 3

   Evidence:

   - *(6th-8th grade units).* CMP makes it explicit to the teacher and the student how math ideas interconnect and build to create a coherent whole. In the beginning of each Teacher Guide, the big ideas of the present unit and the connections to the past and future units are given in a table. Students see the connections directly by doing the "C" (Connection) problems after each Investigation.
   - *(6) Data About Us.* On p. 49, students make a graph of whole numbers vs. the number of factors. This is connecting to concepts studied previously in *Prime Time.*

- *(7) Moving Straight Ahead.* On p. 79h the teacher is shown how similar triangles are related to the slope of a line.
- *(8) Looking for Pythagoras.* On p. 68 and p. 16a, connections between similar triangles, proportionality and the Pythagorean Theorem are made.

Discussion: This standard is fully met.

3. *Does the curriculum enable all students to recognize and apply math in contexts outside math?*

Score: 3

Evidence:

- *(6) Prime Time.* On p. 11, connections to finding prime numbers and the Cray Computer are given. On p. 38, cycles of locusts are connected to common multiples. On p. 60 in Extension problem 17, students have to discuss why a minute is divided into 60 units rather than 61 or 59.
- *(6) Data about Us.* On p. 54, census data are used to make a frequency graph. On p. 84, political campaigns are examples of the convenient interpretation of the word average. Students analyze which candidate is telling the truth.
- *(6) Bits and Pieces I.* The locations of cars in drag racing on a linear race track are related to positions on a number line. Likewise, gauges in water containers are used in measurement studies. In IV6, surveys use percentages. In IV5, probems 42 and 43 use the Dewey Decimal system to study decimals.
- *(6) Shapes and Designs.* On p. 11, polygons are connected to street signs. In IV2, triangles are seen to produce stable structures, such as bridges. On p. 32, measurement error is related to Amelia Earhart's last flight. On p. 8, honeycombs are related to hexagonal tesselations.
- *(6) How Likely Is It?* Probability is used to forcast rain or snow. In IV7, probability is used to study events in genetics, such as having a blue-eyed child.
- *(7) Data Around Us.* In problem 17a, p. 68, students find the average number of miles per day travelled by the Galileo spacecraft.
- *(7) Moving Straight Ahead.* In IV1, the stretching of a spring under weight is a connection to physics.
- *(8) Thinking with Mathematical Models.* Compound interest is an example of a nonlinear relationship.
- *(8) Clever Counting.* Students do combinatorical problems including zip codes, phone numbers, and lock combinations.

Discussion: This standard is fully met.

B11.2 Connection Summary

Question	CMP
Connections 1.	3
Connections 2.	3
Connections 3.	3

Table 9: Summary of NCTM Connections Standard Results

The scores in the table above show that the curriculum fully meets all three of the standards.

B12 Representation Standard

B12.1 Representation Questions and Evidence

Note: In what follows IV1 refers to Investigation 1, IV2 to Investigation 2, etc.

1. *Does the curriculum enable all students to create and use representations to organize, record, and communicate mathematical ideas?*

   Score: 3

   Evidence:

   - *(6) Prime Time.* This unit uses systematic lists, Venn diagrams, square tiles, areas to represent factors, and factor trees for prime factorization.
   - *(6) Data About Us.* This unit uses line plots, bar graphs, frequency graphs, stem-and-leaf plots, pie charts, area models, double stem-and-leaf plots, scatter plots, and coordinate graphs.
   - *(6) Bits and Pieces I.* On p. 5, students make thermometers to measure the percentage of a fund-raising goal that is met. Fraction strips, 100 grids, and area models are used to convey fractions and percent ideas.
   - *(7) Data Around Us.* This unit uses base-10 units, rods, and flats to represent large integers and their relationships.
   - *(7) What Do You Expect?.* This unit uses probability trees, spinners, colored balls, dice, coins, playing cards, thumb tacks (lands point up or down), area models, and number cubes.
   - *(7) Accentuate the Negative.* In problem 3.1, p. 37, colored chips and number lines are used to develop meaning for addition, subtraction, and multiplication with negative numbers.
   - *(8) Clever Counting.* Counting trees and networks are used to illustrate combinatorial problems.
   - *(8) Samples and Populations.* Box and whisker plots, stem-and-leaf plots, scatter plots, histograms, spinners, dice, and random number generators are used to study statistics and probability.
   - *(8) Growing, Growing, Growing.* Tree diagrams are used to illustrate exponential growth or decay.

   Discussion: This standard is fully met.

2. *Does the curriculum enable all students to select, apply, and translate among mathematical representations to solve problems?*

   Score: 3

   Evidence:

- *(6) Data About Us.* Both stem-and-leaf and scatter plots are used for the same data.
- *(6) Bits and Pieces I.* On p. 18e, the teacher is encouraged to help students go back and forth between words and fraction strips to reinforce the meaning of fractions. Teachers are encouraged not to rush to symbols until the meaning is clear.
- *(6) Ruins of Montarek.* Students translate between different views of a solid in order to build the solid. Alternately, they view the solid and fill in the missing parts in 2D views. This is a comprehensive spatial visualization unit.
- *(7) Accentuate the Negative.* On pp. 46–49, students translate between chip boards and number lines to add and subtract positive and negative integers.
- *(7) Stretching and Shrinking.* Students build a device called a rubber-band stretcher to enable them to produce similar figures.
- *(7) Variables and Patterns.* Students go between tables, graphs, equations, and words to discuss, represent, and analyze relationships.
- *(8) Looking for Pythagoras.* In IV2, students translate between dot paper and coordinate grid representations to find areas of plane figures.

Discussion: This standard is fully met.

3. *Does the curriculum enable all students to use representations to model and interpret physical, social, and mathematical phenomena?*

Score: 3

Evidence:

- *(6) Data About Us.* On p. 7 students make frequency graphs of the number of letters in people's names. On p. 54 census data are displayed using frequency graphs.
- *(6) Covering and Surrounding.* In the Unit Project, students build a plan for a park using a proper scale.
- *(6) Ruins of Montarek.* Isometric dot paper and architectural views are used to represent a solid figure.
- *(7) Variables and Patterns.* Students use graphs to extrapolate the relationship outside the data set and interpolation to find a reasonable relationship between the data points.
- *(7) Data Around Us.* In IV1, students use correct measurement units to better interpret large numbers in social situations, such as disaster results such as the Exxon Valdez oil spill.
- *(8) Frogs, Fleas, and Painted Cubes.* Rectangle models are used to come up with algebraic equations for quadratic relationships.

- *(8) Thinking with Mathematical Models.* This entire book is about using graphs, tables, and physical models to describe linear and nonlinear relationships and patterns that represent (as best as one can at this level) real-world situations. Students use these representations to model these situations.

Discussion: This standard is fully met.

B12.2 Representation Summary

Question	Singapore
Representation 1.	3
Representation 2.	3
Representation 3.	3

Table 10: Summary of NCTM Representation Standard Results

The scores in the table above show that the curriculum fully meets all three of the standards.

B13 Alignment with NCTM Principles

B13.1 Principles Questions and Evidence

1. The Equity Principle. *Excellence in mathematics education requires equity-high expectations and strong support for all students. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students.*

1. Does the curriculum provide materials and suggestions to the teacher for individualizing instruction?

0	1	2	3
All students do the same tasks	Low	Medium	High number of materials and tips for individualizing

Score: 3

Discussion: This principle is fully met. The curriculum gives Extension problems that can be used for abler students. Extra challenges are given to the teacher for those that finish early. The teacher is told how to alter the assessments for inclusion students to individualize instruction for them. Teachers are encouraged to find out by group work who understands and who is confused. The Embedded Assessments focus on what students know and feeds back into the planning of lessons.

2. Are the curriculum materials likely to be interesting, engaging, and effective for girls and boys?

0	1	2	3
No sensitivity to gender issues	Low	Medium	High sensitivity to gender issues

Score: 3

Discussion: This principle is fully met. Equal attention is paid to boys and girls in the curriculum. The materials should be equally beneficial to boys and girls.

3. Are the curriculum materials likely to be interesting, engaging, and effective for underrepresented and underserved students (e.g., ethnic, rural, with disabilities)?

0	1	2	3
No sensitivity to underrepresented and underserved students	Low	Medium	High sensitivity to underrepresented and underserved students

Score: 3

Discussion: This principle is fully met. For United States students, this curriculum does a thorough job of including problems with stories about different parts of the country, both urban and rural and includes the groups outlined above.

4. Are the curriculum materials likely to be interesting, engaging, and effective for mathematically capable students?

0	1	2	3
No	Low	Medium	Highly interesting,
Level too low			engaging, and effective
even with the			without any extension
extension			materials
materials			

Score: 2

Discussion: This principle is adequately met. The materials, even though beneficial to these students, is not enough. More problems at different levels should be included to help the teacher individualize the instruction. In CMP classooms we visited, the teachers brought in more challenging problems to supplement the curriculum. The combination worked well. CMP helped them slow down to make sure the concept was totally understood for all and they supplemented for the most capable students.

2. The Curriculum Principle. *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades. The interconnections between the mathematics strands should be displayed prominently in the curriculum and in instructional materials and lessons. A coherent curriculum effectively organizes and integrates important mathematical ideas so that students can see how the ideas build on, or connect with, other ideas, thus enabling them to develop new understandings and skills. The curriculum should help teachers understand the mathematics that has been studied at previous levels and what is the focus at successive levels. A well-articulated curriculum gives teachers guidance regarding important ideas and major themes and depth of study warranted at particular times and when closure is expected for particular skills or concepts.*

1. Is the mathematics curriculum coherent?

0	1	2	3
No	Low	Medium	Very much so

Score: 3

Discussion: This principle is fully met. The curriculum is very coherent for the teacher. In the Teacher's Guide for each unit, the "Connections to Other Units" gives a table with the three columns "Big Idea," "Prior Work," and "Future Work." This includes work in other mathematical strands. The curriculum is also explicitly coherent for the student. At the end of each Investigation, there are three sets of problems, called ACE. The "A" problems apply to the current topic, the "C" are the connection problems that connect the current topic to other strands and to concepts previously learned in the current strand, and the "E" problems are extension problems.

2. Does the curriculum focus on important mathematics?

0	1	2	3
No	Low	Medium	Very much so

Score: 3

Discussion: This principle is fully met. All the standards are represented in this curriculum. The curriculum focusses on the understanding of the big ideas in these strands.

3. Is the mathematics curriculum well-articulated across the grades (6-8)?

0	1	2	3
No	Low	Medium	Very much so

Score: 3

Discussion: This principle is fully met. In the Teacher's Guide for each unit, articulation is given in a table called "Connections to Other Units." This table shows how the big ideas of the present unit were built from prior units, and how they are building to future units. More articulation is given to the teacher in the "Getting to Know Connected Mathematics" pamphlet. This gives the scope by strands, the recommended order by grade and a flowchart of the connections of the units in each grade.

3. The Teaching Principle. *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well. To be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks.*

1. Does the curriculum help the teacher to deeply understand the mathematics he or she needs to teach the students?

0	1	2	3
No	Low	Medium	Very much so

Score: 3

Discussion: This principle is fully met. In the first four or five pages of the Teacher's Guide for each unit, the "Mathematics in the Unit" explains the mathematics to the teacher. Helpful hints also appear in the "Teaching the Investigation." Answers to the problems often have worked out solutions in the Teacher's Guide.

4. The Learning Principle. *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. A major goal of school mathematics is to create autonomous learners who when challenged with appropriate tasks are confident in their ability to tackle difficult problems, eager to figure things out on their own, are flexible in exploring mathematical ideas and trying alternative solution paths, and are willing to persevere.*

1. Does the curriculum promote learning with understanding?

0	1	2	3
No	Low	Medium	Very much so

Score: 3

Discussion: This principle is fully met. The emphasis in this curriculum is that students should understand the conceptual development of the mathematics.

2. Does the curriculum encourage students to be autonomous learners?

0	1	2	3
No	Low	Medium	Very much so

Score: 3

Discussion: This principle is fully met. Students are encouraged to take charge of their own learning. They choose appropriate strategies and tools. They explain, analyze, reflect on, and refine their mathematical work. They decide when using a calculator is appropriate. This mode of operation is intended to be carefully monitored by a thoughtful and well-qualified mathematics teacher.

5. The Assessment Principle. *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students. Assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions.*

1. Does the curriculum include and encourage multiple kinds of assessments (e.g. performance, formative, summative, paper-pencil, observations, portfolios, journals, student interviews, projects)?

0	1	2	3
No	Low	Medium	Very much so

Score: 3

Discussion: This principle is fully met. These kinds of assessments are made explicit in each unit of the curriculum. Checkups, Quizzes (A and B), Unit Tests, Question Banks, Self-Assessment Forms, Student Notebook Checklists, and Partner Quizzes are all included with the curriculum. Students keep Journals of their work.

2. Does the curriculum provide well-aligned summative assessments to judge a student's attainment?

0	1	2	3
No	Low	Medium	Very much so

Score: 3

Discussion: This principle is fully met. Well-aligned summative assessments are given at the end of each unit for the teacher's use.

6. The Technology Principle. *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving. Technology should not be used as a replacement for basic understandings and intuitions; rather it can and should be used to foster those understandings and intuitions.*

1. Does the curriculum use technology as a tool for learning and doing mathematics?

0	1	2	3
No	Low	Medium	Very much so

Score: 3

Discussion: This principle is fully met. CMP teaches students *when* and *how* to use calculators and graphical calculators. Students also use a LOGO program called Turtle Math and its Shape, Grid, and Scale Tools to study geometry. The use of graphical calculators starts in 7th grade.

B13.2 Principles Summary

Principle	CMP
Equity 1.	3
Equity 2.	3
Equity 3.	3
Equity 4.	2
Curriculum 1.	3
Curriculum 2.	3
Curriculum 3.	3
Teaching 1.	3
Learning 1.	3
Learning 2.	3
Assessment 1.	3
Assessment 2.	3
Technology 1.	3

Table 11: Summary of Alignment with the NCTM Principles

The scores in the table above show that the curriculum fully meets twelve of the thirteen principles and adequately meets one of them. The lower score was given because the materials will have to be supplemented to keep the interest of the more capable students.

B14 Table of CMP Results

Standard	Questions Where Deficiencies Are Revealed
Number	1,8,9,12
Algebra	none
Geometry	8
Measurement	2
Data and Probability	7,8
Problem Solving	none
Reason and Proof	none
Communication	none
Connection	none
Representation	none
Principle	
Equity	4
Curriculum	none
Teaching	none
Learning	none
Assessment	none
Technology	none

Table 12: Connected Mathematics Program Summary