D. Project Description

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We propose a research project whose objective is to develop a thorough understanding of three-dimensional water waves of finite amplitude. Our long-term goal is to develop a practical theory for inviscid, three-dimensional water waves of finite amplitude. During the tenure of the grant, we intend to construct the basic building blocks of such a theory in the fully nonlinear, three-dimensional regime. We also intend to relate the detailed dynamics of nonlinear, three-dimensional water waves to some of the ocean-wave transport models that are used by oceanographers today. Here our goals are to describe large amplitude nonlinear waves with more accuracy, and to find a rigorous mathematical basis for the validity of the transport models. Other projects that are naturally related to and resulting from our main proposal include numerical boundary spectral methods for water waves and some new inverse problems involving water waves.

Our focused research group consists of eight mathematicians and scientists, whose areas of expertise are very diverse: physical experiments, perturbation theory, scientific computation, algebraic geometry and mathematical analysis. In addition, our research group has involved people at all levels of seniority, from senior faculty through postdoctoral researchers, graduate students and advanced undergraduates. Our group has been successfully working together for more than one year, and we have held six meetings during that year. An invigorating part of our collaboration is the diversity of our areas of expertise and points of view. A sample of recent results by members of this group includes: a rigorous existence theorem for three dimensional doubly-periodic water wave patterns, experimental observations of doubly-periodic wave patterns both in shallow water and in deep water, numerical computations (using boundary spectral methods) of doubly periodic wave patterns in water of any depth, and asymptotic analysis of such patterns in either the KP (i.e., shallow water) or NLS (deep water) regimes.

Our specific scientific goals for future research include: (1) the existence and stability of three-dimensional, doubly-periodic, traveling water-wave patterns, through the full range of depths; (2) the prevalence of hexagonal, rectangular or crescent-shaped waves (or other multiply periodic wave patterns) among ocean waves; (3) the long-wave and modulational descriptions of water waves, and the subsequent stability analyses that are feasible in these cases; (4) the design and implementation of algorithms to make practical use of exact solutions of asymptotic models in shallow and deep water; (5) the relation between the detailed dynamics of three-dimensional, nonlinear waves and some commonly used ocean-wave transport models; and (6) the impact of a detailed local description of nonlinear wave dynamics on these transport models, in the presence of large amplitude nonlinear waves or under conditions of nonlinear wave focusing. The diversity of our research group and the multiplicity of our areas of expertise is an innovative aspect of our collaboration, and we expect that it will play a major role in our work.

An outline of this section is as follows. Section C.1 contains an explanation of the scientific context and timeliness of the project, and the justification for why our group effort is necessary. In Section C.2 is a description of the proposed research. Section C.3 contains a timeline. Section C.4 presents plans for disseminating the results, and Section C.5 contains results of prior, related NSF support. A description of new modes of training students and postdoctoral researchers is included in Section C.6. Section C.7 is a description of planned workshops with a list of tentative participants. Finally, a management plan is given in C.8.

D.1 The scientific context and timeliness.

In this section, we provide a historical context of the general study of surface waves that has led to the formation of our group (C.1.a), a theoretical framework for inviscid surface waves (C.1.b), and a discussion of why our group effort is necessary (C.1.c).

D.1.a. Scientific and Historical Context

The study of water waves has been around for as long as ocean navigation has been around. Serious development of a theory for (inviscid) water waves dates back at least to Stokes (1847), who studied the boundary-value problem we refer to as Euler's equations. Its formulation is given in (C.1.b). Two recurring questions in the development of the theory have been: ***Does a stable wave of permanent form exist? If so, can one use such waveforms to build a practical, predictive model of water waves that is both accurate and robust?*** These two related questions have guided much of the past study of water waves. We explain below how they also guide the study we propose for our FRG and why we expect to make substantial progress on these long-standing questions.

Over the years, investigators have hypothesized various wave forms as the candidate for the basic building block of a practical theory. In 1847, Stokes wrote down the solution of the linearized Euler's equations. Then he considered this to be the first term in an asymptotic expansion, and carried the expansion to three orders to find the first nonlinear correction to the linearized dispersion relation. He apparently had in mind that a uniform plane wave (or wavetrain) is the permanent form wave being sought. His ideas were given a sound mathematical basis by Nekrassov (1921), Levi-Civita (1925), and Struik (1926), who constructed convergent series solutions of the governing equations for spatially periodic, traveling waves of permanent form. The wave form is a uniform plane wave, with a 1-D surface pattern and 2-D fluid motion; we call it a 2-D wave in this proposal.

These series expansions converge for "small enough" wave amplitudes, so the waves have finite but rather small amplitudes. Later, investigators improved on these ideas, so larger amplitudes were included in the theory. Krasovskii (1960, 1961) proved existence of permanent, periodic, 2-D waves with the only restriction being that the "peak angle" could not exceed $\pi/6$. Amick & Toland (1981) have a single theory that includes earlier results by many people. Their theory also proves the existence of a periodic, 2-D wave of any wavelength, and with a peak angle anywhere from 0 up to the same maximum, $\pi/6$. As the wavelength $\to \infty$, their work gives back solitary waves with the same properties. Amick & Toland's work was for gravity waves. Beale (1979), and Amick & Kirchgassner (1989) included surface tension.

So, uniform trains of 2-D, periodic, plane waves *exist*. However, Benjamin & Feir (1967) showed that these waves are modulationally *unstable*. At about the same time, Zakharov (1968) also showed that such waves are unstable, and he derived the Nonlinear Schrödinger (NLS) equation (given in C.1.b) to describe the instability in 1-D.

Whitham (1967) showed that the Benjamin-Feir instability ceases to function for $kh < 1.36$, where k is the wavenumber and h is the water depth. Here is where the bifurcation between "shallow water" and "deep water" appears. In the remainder of this proposal, "shallow water" requires $kh < 1.36$; "deep water" requires $kh > 1.36$. The following discusses each possibility separately.

Whitham's analysis suggests that stable wave forms might exist in shallow water, since the Benjamin-Feir instability does not function in that regime. Other instabilities might arise, but the Korteweg-deVries (KdV) equation describes wave propagation approximately for 2-D waves in shallow water, and it has periodic and solitary wave solutions that are stable (within that equation). Further, experiments (*e.g.*, Goring & Raichlen, 1980) suggest that uniformly traveling plane waves are stable in shallow water (that is, for $kh < 1.36$), and coastal engineers (*cf*. Wiegel, 1960, and Dean, 1974) use both periodic 2-D waves and solitary 2-D waves in practice. For their purposes, a permanent form wave (whether obtained analytically, numerically or in experiments) that is stable over a long enough time-scale and length-scale is adequate for practical applications.

What about 3-D, permanent form waves in shallow water? The Kadomtsev-Petviashvili (KP) equation (given in C.1.b) describes 3-D waves in shallow water. Both KdV and KP have N-phase solutions for arbitrary N. In particular, typical 2-phase solutions of KP are spatially periodic, genuinely 3-D, traveling waves of permanent form, with finite amplitudes. We refer to them as "hexagonal waves". This family of solutions includes the 2-D (cnoidal) waves as special cases. Experiments (Hammack, *et al.*, 1989, 1995) show that there are actual hexagonal water waves like these KP solutions (see Fig. 2). The experiments further suggest that these hexagonal waves are stable and have applications to coastal processes (Hammack, *et al.*, 1991, and in preparation). Thus, in shallow water, 3-D permanent form waves exist and survive for long enough times to be useful for modeling.

KdV and KP are only approximate equations, so solutions of these equations do not prove that the full equations admit corresponding solutions. Fortunately, Craig & Nicholls (2000) proved exactly this, with nonzero surface tension present. (We note that the question of existence for zero surface tension is an interesting, open, mathematical question. It is not necessarily relevant for experiments or ocean waves.) Numerics by Nicholls (1998) show the striking hexagonal character of solutions. Thus, mathematical *existence* is no longer an issue for hexagonal-type waves in shallow water. The experiments by Hammack *et al.* (1989, 1995) do not imply that the hexagonal waves are stable, because the experiments have only a finite duration (that is, the instability might be too slow to be seen in the tank of finite length.) Thus, Craig & Sulem are proposing herein to explore the *stability* of hexagonal waves in shallow water within Euler's equations. Further, if the hexagonal waves are stable, what about N-phase KP solutions with $N > 2$? Typically, N-phase KP solutions do not have permanent form, in any coordinate system, but they have a relatively simple description, and they might be stable. As part of this proposal, Deconinck & Segur will explore more complicated solutions of the KP equation, for shallow water.

In summary, it is likely that in shallow water, the original question, **Do stable waves of permanent form exist?** has a positive answer. One objective of our FRG is to begin to use these stable wave forms to build a practical model of wave propagation in shallow water.

In deep water ($kh > 1.36$), the situation is more ambiguous and perhaps more interesting. Here uniform plane waves are known to be unstable. Zakharov and Shabat (1972) showed that for plane waves (i.e., with 1-D surface patterns), "envelope solitons" are stable within the context of the 1-D NLS approximation. This was confirmed experimentally by Yuen and Lake (1975) and Hammack (1979). But Zakharov and Rubenchik (1974) also showed that these 1-D wave packets are unstable to long transverse perturbations; ie., they are stable in 1-D but unstable in 2-D. This was also confirmed experimentally by Hammack (1979).

Computations by Nicholls (1998), along with earlier computations by Roberts (1983), Bryant (1985), Roberts and Peregrine (1983) and others, suggest that there are traveling waves of permanent form in deep water with genuinely 2-D surface patterns, and so with 3-D velocity fields. Craig and Nicholls (2000) gave these formal calculations a firm theoretical basis when they proved that in the presence of both gravity and surface tension, Euler's equations admit traveling waves of permanent form, with surface patterns that are genuinely 2-D and spatially periodic, in water of any depth. Thus, waves of permanent form with 3-D velocity fields *exist* in deep water.

The form of these waves is controversial. The computations of Nicholls indicated that as the water depth increases, waves that were hexagonal in shallow water should become more rectangular in deep water. However, Bridges *et al.* (2001) have computed hexagonal wave patterns in deep water. Experiments by Kimmoun et al (1999) show traveling waves with apparently permanent form and genuinely 2-D periodic surface patterns in deep water. Some of their experiments show rectangular patterns (as predicted by Nicholls), but other experiments show different quadrilateral patterns. Recent experiments by Hammack &Henderson (HH), showing waves of permanent form in deep water, are shown in Figure 1. Here the surface patterns are rectangular, as predicted by Nicholls. The experimental facility developed by HH permits precise, quantitative, temporal and spatial measurements of these waves. They will use it (indeed, they have already begun to do so, as discussed below) to settle certain controversies about the precise form of these wave patterns. That is one

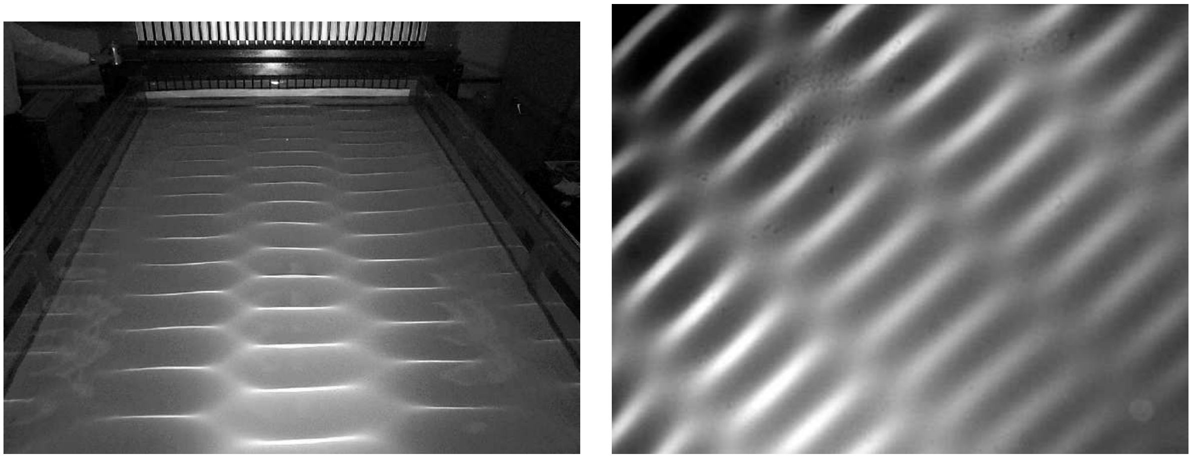

Figure 1: *Photographs of 3 and 4 Hz waves in deep water, showing apparently hexagonal and rectangular wave patterns (Hammack & Henderson)*

objective of our FRG.

The stability of these 3-D waves of permanent form is also ambiguous. The fact that these waves are seen in experiments by Kimmoun *et al*, as well as in those by HH, indicates that the waves enjoy some measure of stability: if the waves were unstable enough, we would not even be able to see them experimentally. Even so, Carter (2001) recently showed that within the context of the 2-D NLS model, these waves should be unstable in deep water. This is another paradox that needs to be resolved.

In addition to the rectangular waves shown in Figure 1, crescent-shaped waves also occur as spatially periodic wave patterns in deep water. First identified by Su (1982) and Su *et al.* (1982), the appearance of crescent waves is controversial. The original experiments by Su *et al.* show some crescents that face forward and others that face backwards. In somewhat different experiments, Collard & Caulliez (1999) found that crescents always face forward. Annekov & Shrira (2000) assumed that crescents always face forward, and developed a theoretical model to explain this phenomenon. However successful this model is, it does not explain the observations of Su *et al.*: that crescents can face either way. We propose to address this issue in our FRG, as we discuss in more detail in section C.2.b. As with the rectangular waves, the breadth of our group allows us to study the problem analytically, numerically, and experimentally.

To summarize, the situation is quite unclear in deep water. Traveling waves of permanent form, with spatially periodic, 2-D surface patterns, are known to exist in deep water. There is controversy about the precise form of these waves, and there is more controversy about their stability. The advantage of the breadth of our FRG is that we can resolve these issues within our group. By addressing the same problem with a variety of tools (analysis, approximations, numerical computations, experiments), we can resolve these paradoxes. Resolving the paradoxes about waves of permanent form in deep water is one of our highest priorities.

How do these mathematical results fit into the present operational wave-forecasting models, which are spectral and phase-averaged? For waves outside the surf zone, these models, which are based on the ideas developed for "WAM" (WAMDIG 1988, Komen et al. 1994), solve an equation for spectral energy balance which is posed in terms of distributions of complex wave amplitudes as a function of wave vector. Wave propagation is considered to be principally linear, and the models include refraction and straining of the wave field due to temporal and spatial variations of the mean water depth and mean current (tides, surges, etc), and wave growth and decay due to the actions of the wind, nonlinear four-wave resonant interactions, dissipation ("white-capping") and bottom friction. These effects come into the balance as source terms, interaction terms and dissipation terms, and in practice in actual forecasting they are frequently updated (Tolman & Chalikov 1996). For waves in shallow water, forecasters also use spectral, phase-averaged models such as SWAN (Padilla-Hernandez et al 1998). These models are presently being improved to include triad interactions in the nonlinear interaction term, wave interaction with currents, phase-resolving capability, and coupling to data assimilation techniques. We propose to investigate which contributions our group can make to further improve this class of operational models, based on our results on the detailed local and nonlinear dynamics of three-dimensional water waves. For example, the present operational models are unable to accurately predict events in which large amplitude waves are generated through focusing or through other phenomena involving nonlinear superpositioning (Wyatt 2001), sometimes known as 'rogue waves'. Forecasting goals would be to estimate the likelihood of such events, over a range of scales of severity, from the probability of a given percent increase in amplitude of individual waves in a wavefield, to *e.g.* estimates of the largest 100 year North Sea storm-generated wave. Predictions such as these have a potential impact on actual sea-state forecasting (P. Janssen 2001) and the routing and scheduling of shipping, as well as having design implications for offshore platforms (Bateman, Swan & Taylor 2001). Our intention is to investigate the impact of a detailed local description of nonlinear wave dynamics on a class of operational models, in the presence of large amplitude nonlinear waves or under conditions leading to nonlinear wave focusing.

D.1.b. Governing Equations

The governing equations for the irrotational flow of an inviscid, incompressible fluid with a free surface are given below. These include the full water-wave problem, which we refer to as Euler's equations, as well as the new Zakharov-Craig-Sulem (ZCS) formulation of that problem. Then we list some approximate models that are relevant to our work.

Inviscid surface waves in arbitrary depth h that are fully three-dimensional and fully nonlinear are described by Euler's equations for the irrotational velocity field $\mathbf{v}(x,y,z,t) = \nabla\varphi(x,y,z,t)$ and the free surface displacement $\eta(x,y,t)$:

$$\Delta\varphi = 0 \qquad \text{in} \qquad -h < z < \eta(x,y,t), \quad -\infty < x,y < \infty \tag{1a}$$

$$\frac{\partial\eta}{\partial t} + \nabla\varphi\cdot\nabla\eta - \frac{\partial\varphi}{\partial z} = 0 \quad \text{on} \quad z = \eta(x,y,t) \tag{1b}$$

$$\frac{\partial\varphi}{\partial t} + \frac{1}{2}|\nabla\varphi|^2 + g\,\eta - T\,\nabla\cdot\left[\frac{\nabla\eta}{\sqrt{1+|\nabla\eta|^2}}\right] = 0 \quad \text{on} \quad z = \eta(x,y,t) \tag{1c}$$

$$\frac{\partial\varphi}{\partial z} = 0 \quad \text{on} \quad z = -h. \tag{1d}$$

Here T is the coefficient of surface tension, and g is the acceleration of gravity.

Zakharov (1968) showed that these equations can be written in Hamiltonian form. Craig & Sulem (1993) introduced the Dirichlet-Neumann operator, $G(\eta)$, into Zakharov's Hamiltonian formulation and investigated the time-dependent two-dimensional version of the resulting equations. The operator takes Dirichlet data at the fluid surface to Neumann data at the surface. Let $\xi(x,y,t) = \varphi(x,y,\eta(x,y,t),t)$ with surface normal $N_\eta = (-\nabla_\perp\eta, 1)^T$. Then

$$G(\eta)\xi = N_\eta\cdot\nabla\varphi|_{z=\eta(x,y,t)}. \tag{2}$$

Then the boundary conditions at the free surface are

$$\frac{\partial\eta}{\partial t} = G(\eta)\,\xi, \tag{3a}$$

$$\frac{\partial\xi}{\partial t} = -g\,\eta - \frac{1}{2|N_\eta|}\Big(|\nabla_\perp\xi|^2 - (G(\eta)\xi)^2 - 2(G(\eta)\,\xi)\nabla\xi\cdot\nabla\eta + |\nabla_\perp\xi|^2|\nabla_\perp\eta|^2 - (\nabla_\perp\xi\cdot\nabla_\perp\eta)^2\Big). \tag{3b}$$

Thus, the ZCS formulation is a surface-integral formulation of the full water wave problem.

Several approximate models can be obtained from (1). Of particular interest here are the KP equation for waves in shallow water, and the nonlinear Schrödinger (NLS) equation and its generalizations for waves in deep water. The (dimensionless) KP equation for gravity-dominated waves that propagate mainly in the x-direction in shallow water is

$$(u_t + u\,u_x + u_{xxx})_x + u_{yy} = 0. \tag{4}$$

This equation can be viewed as a two-dimensional generalization of the famous Korteweg-de Vries (KdV) equation, to which it reduces if $u_{yy} \equiv 0$. Its derivation follows the same lines as the derivation of the KdV equation (i.e., small amplitude waves in shallow water), but the KP equation allows these waves to vary slowing in the y-direction, while the KdV equation does not. (For a full discussion, see Segur & Finkel, 1985, among others.) The fact that (4) admits 2-D surface patterns is crucial to describing the hexagonal waves observed by Hammack *et al.* (1989, 1995).

The KP equation assumes that the y-wavelength of the waves is long compared to the x-wavelength. It is completely integrable and has a countably infinite family of exact periodic or quasi-periodic solutions that are expressed in terms of Riemann theta functions with N phases (Krichever, 1977). The one-phase solutions correspond to 1-D, “cnoidal” wave trains with one real frequency and wavelength, originally found by Korteweg & DeVries (1895). Two-phase solutions typically correspond to a genuinely 3-D wave field with two real frequencies and wavelengths, and so on for more phases. The effort to turn these solutions into effective tools for comparison with experimental or numerical data is ongoing (Segur & Finkel, 1985; Bobenko & Bordag, 1989; Dubrovin *et al.*, 1997; Deconinck & Segur, 1998; Deconinck & Van Hoeij, 2001). A movie of the computed, exact, three-phase solutions by Dubrovin *et al.* is available at `http://amath.colorado.edu/activities.kp`.

In water of fixed but arbitrary depth, a group of nearly monochromatic waves of small amplitude, propagating primarily in the x-direction but with modulations in both x- and y-directions is governed by equations derived by Benney & Roskes (1969), Davey & Stewartson (1974), and Djordjevic & Redekopp (1977). These are usually called the Davey-Stewartson (DS) equations:

$$\begin{aligned} i\,A_t + \zeta\,A_{xx} + \mu\,A_{yy} &= \chi\,|A|^2 A + \chi_1\,\Phi_x\,A, && \text{(5a)} \\ \beta\,\Phi_{xx} + \Phi_{yy} &= -\beta_1\,(|A|^2)_x, && \text{(5b)} \end{aligned}$$

for the complex amplitude A of the wave envelope. Here, the coefficients are determined by the underlying wavetrain, Φ is a velocity potential for the mean flow that is induced by the wavefield, and the spatial coordinates are referenced to a frame moving with the wavetrain at its group velocity. In the case of water of infinite depth, the mean flow disappears at this order. If gravity dominates surface tension, then (5) reduces to

$$i\,A_t - A_{xx} + A_{yy} = |A|^2 A, \tag{6}$$

which is a version of the two-dimensional nonlinear Schrödinger (NLS) equation. The equation arises in many physical contexts, so it has been studied in detail (*cf.* Sulem & Sulem, 1999). For our purposes, it is important to note that the equation admits periodic solutions in the form of elliptic functions. These solutions describe modulations of an underlying wavetrain in the direction of propagation (x), in the transverse direction (y), or in an oblique direction. These modulations of the wave envelope provide a second period of the wave pattern, corresponding to the doubly-periodic surface patterns seen in our recent experiments and in Figure 1.

As a result of our recent group meetings, we have derived a coupled generalization of (6) without surface tension, which, in the deep water limit and with two coupled waves reduces to

$$\begin{aligned} i\,(A_t + \mathbf{C_g}\cdot\nabla A) + \epsilon\left(\alpha_A\,A_{xx} + \beta_A\,A_{yy} + \gamma_A\,A_{xy} + \lambda_A\,|A|^2 A + \chi_A\,|B|^2 A\right) &= 0, && \text{(7a)} \\ i\,(B_t + \mathbf{C_g}\cdot\nabla B) + \epsilon\left(\alpha_B\,B_{xx} + \beta_B\,B_{yy} + \gamma_B\,B_{xy} + \lambda_B\,|B|^2 B + \chi_B\,|A|^2 B\right) &= 0, && \text{(7b)} \end{aligned}$$

for the complex envelopes, A and B, of two wavetrains at arbitrary angles. These equations have been written down by Benney & Newell (1967) (with arbitrary coefficients) and have been derived in 1-D by Pierce & Knobloch (1994) (see also Knobloch and Pierce (1998)) for waves on water of finite depth. The equations (7a-b) have a natural generalization to any number of coupled waves. Such generalizations will also be investigated. We will compare and contrast (7) with (6), with experimental data, and with Nicholl's computations on the full Euler equations under this grant.

There is a huge literature comprising numerical and analytical investigations of 3-D wavefields in deep water. Rather than review it here, we refer to reviews given in, for example, Dias & Haragus-Courcelle (2000) or Kimmoun, *et al.* (1999). In addition to these recent papers, there has been an edition of the *Eur. J. Mech. B/Fluids*, 18(3), (2000) that contains papers from a recent conference on 3-D waves in deep water (with or without the effects of wind).

D.1.c. Justification for the Group Effort

We now consider the "timeliness" of our FRG proposal, and ask two questions: What has changed that makes ***now*** the right time for our work? Why are the people in our FRG the right people to do the work? The answer to both questions is that several pieces of the puzzle have recently fallen into place, and our FRG capitalizes on them. Below is a list of some recent developments that make ***now*** the right time for this proposal.

1. Craig & Nicholls (2000) proved the existence of genuinely 3-D wave patterns of permanent form. These wave forms have spatially periodic surface patterns, and finite amplitudes. In shallow water, they look very much like the KP solutions that inspired this work. So the question of existence is answered for these wave patterns. ***This result is new, so our study could not have taken place even 3 years ago***.

2. Hammack & Henderson now have a functioning laboratory in which they are creating and measuring genuinely 3-D wave patterns. The controls in this lab are much more precise than what existed in earlier labs used by Hammack or presently being used by others. (Photographs of seemingly hexagonal shaped waves and rectangular waves are available at http://www.math.psu.edu/dmh/FRG.) ***This facility is new, so our study could not have taken place 3 years ago***.

3. Experimental observations of spatially periodic, 3-D wave patterns that propagate as waves of approximately permanent form in deep water are new. First observed by Kimmoun et al (1999), these waves have now been observed by HH as well (*cf.* Figure 1). The experimental precision available in their laboratory makes their facility ideal for studying these waves. ***Both the observations and the facility are new, so our study could not have taken place 3 years ago.***

4. Nicholls (1998) now has a numerical code to find traveling waves of permanent form in the full Euler's equations, instead of in approximate models (like NLS, KP, or the DS equations). Because HH are now conducting experiments on bi-periodic wave patterns, Nicholls can now look for the observed patterns in his computations. ***This code is new, so our study could not have taken place 3 years ago***.

5. The recent experimental results of HH motivated Carter (2001) to examine the stability of 3-D periodic waves of permanent form in deep water, within the context of (6). His work predicts that these waves are unstable; it also predicts the mode of instability and the growth rate. ***These predictions of instability are new, so our study could not have taken place 3 years ago.***

6. Carter now has working numerical codes for (6), and for higher order generalizations of (6), given by Dysthe (1979), and Trulsen & Dysthe (1996). He can test various initial conditions that HH provide him from experimental measurements. ***His codes are new*** (although the ideas in them are not so new), ***so our study could not have taken place 3 years ago***.

7. Annekov & Shrira (2000) and Craig (2001) have analyzed two different mathematical models of crescent-shaped waves in deep water. Even so, the phenomenon is not yet fully explained. ***These analyses are new, so our study could not have taken place 3 years ago.***

8. It is known that the KP equation admits N-phase solutions. For $N = 2$ or 3, these solutions are available with computer programs by Dubrovin, Flickinger, & Segur, (1997). For arbitrary N, Bordag & Bobenko, (1989) have also computed some solutions with a different algorithm. Now Deconinck is developing a code for symbolic computation with Riemann surfaces of any genus. His code will be publicly available to users. So in the near future, he will be able to produce N-phase all such KP solutions on demand. ***Deconinck's code is nearing completion, so our study could not have taken place 3 years ago***.

In summary, our FRG consists of people with a variety of tools, all to attack the same question: ***Do waves exist that propagate with permanent form over large enough times and distances to act as building blocks for a model of ocean waves?*** Each of us has been working on problems near this problem for years, so our tools have been evolving. It happens that several things have worked out recently, in several aspects of our work, that make **now** the right time for this group to get together for a concerted attack on this problem. In fact, our group has begun collaboration over the last year and met six times. Some of the results from our work are included in the discussion below.

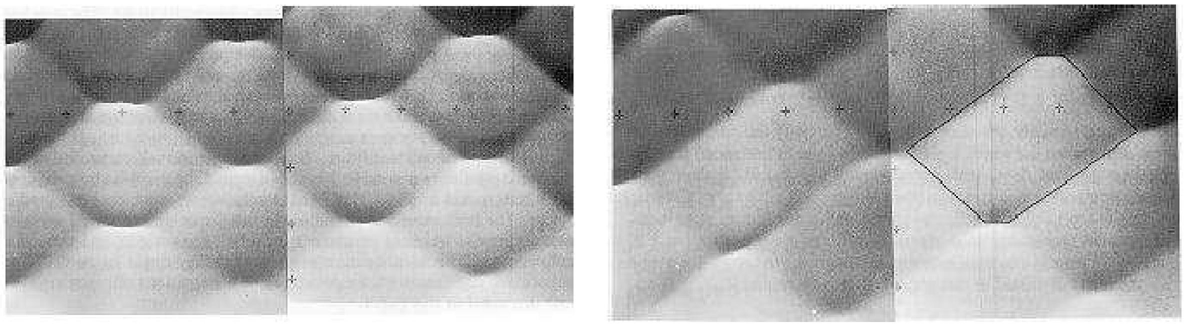

Figure 2: *Overhead photographs of waves in shallow water, showing hexagonal wave patterns (Taken from Hammack* et al. *1995.)*

D.2 Specific problems

Here we discuss specific problems our group will investigate.

D.2.a. Bi-periodic Traveling Waves

Two-dimensional surface patterns that exhibit two spatial periodicities abound in both shallow and deep water and are quite striking to the eye. These waves have been proven to exist in Euler's equations with arbitrary depth (Craig & Nicholls, 2000). They have been observed and studied using approximate models and in experiments in shallow water (Hammack, *et al.*, 1989, 1995), and have now been observed in preliminary experiments in deep water (HH, unpublished). All of this work shows that there are several issues to resolve as we discuss below. To address these issues, we will compare results in shallow and deep water and will compare theory and experiments.

The effects of depth: The numerical experiments of Nicholls & Craig (2000) using the full Euler's equations have shown wavefields that are bi-periodic in space and have qualitatively different signatures depending on depth. This is illustrated in Figure 3. These depth-dependent numerical results are in qualitative agreement with exact solutions of asymptotic models for shallow and deep water waves and with experiments using shallow and deep water waves. An example from experiments of patterns that appear in shallow water are shown in Figure 2, which shows the hexagonal structure of patterns in shallow water.

Recent experiments by HH of wave patterns in deep water are shown in Figure 1. In the first experiment we view the surface obliquely and observe apparently *hexagonal* patterns. In the second experiment we view the surface from overhead and at 45 degrees to the direction of propagation. Here we observe the *rectangular* pattern of off-set crest-trough with its signature nodal lines of zero amplitude. These results are typical of all our experiments in deep water. How do we reconcile these two different views? Our research group met at the Fields Institute in Toronto during June 2001 during which time we addressed this question.

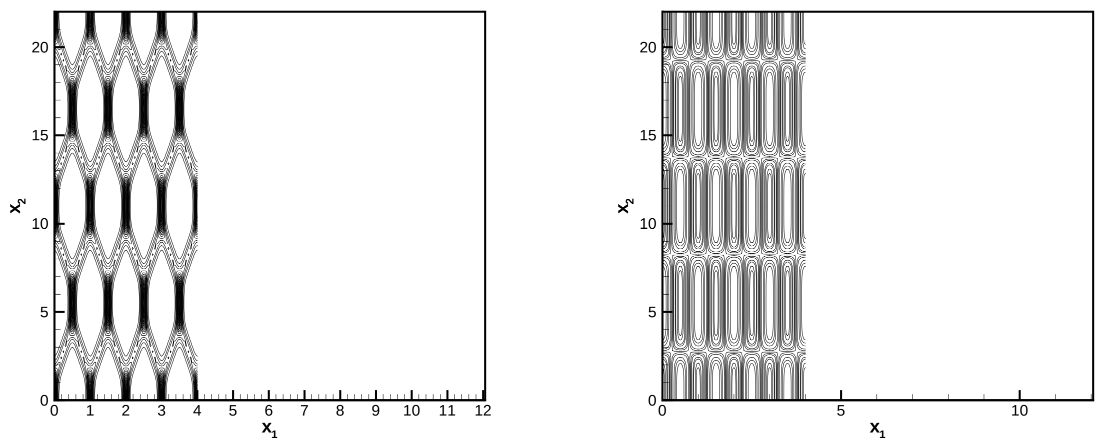

Figure 3: *Contour plots of two traveling-wave solutions to Euler's equations in shallow water (left) and deep water (right) (Nicholls 2001)*

A measurable difference between the two patterns is that time series obtained in the hexagonal patterns may exhibit either one or two temporal periods, depending on the location of the wave gage within the pattern, while time series obtained in the rectangular pattern will exhibit either one temporal period or will be a constant (if taken in the nodal line). Measurements of time series in shallow water by Hammack *et al* (1995) confirm this temporal behavior for the waves depicted in Figure 2. Recent wave-gage measurements by HH show that the temporal behavior of the wavefields in both views of Figure 1 is consistent with a rectangular pattern; that is, the time series show only one period, except in the nodal line where the time series is essentially a constant. In Toronto, we were able to further convince ourselves that the deep-water patterns are indeed rectangular, by considering ray-tracing arguments of the light obtained from the "hexagonal" view in Figure 1.

The issue is confounded by recent computations by Bridges *et al* (2001), who find hexagonal patterns in deep water for interacting wavetrains, in some parameter regimes of nonlinearity and aspect ratio. Thus we propose to consider further the possibility of hexagonal patterns in deep water. In particular, we will use a theoretical solution corresponding to hexagonal patterns as input to numerical experiments of Euler's equations, to the approximate models, and as input at the wavemaker in the experiments. Additionally, we will peruse parameter space in the physical experiment to find regimes in which hexagonal wave patterns may be observed in the physical system.

Exact solutions to an approximate model, (6), do describe the rectangular patterns. However, it is unlikely that hexagonal patterns will be observed in this equation because it assumes a separation of length scales that does not exist in the hexagonal pattern. Further, this equation is not a model for interacting wavetrains. Thus, is NLS an appropriate approximate model for waves in deep water, or is there a new approximate model that is more appropriate? Deconinck, Henderson & Segur will investigate this problem in general. At present, our group is now also considering coupled NLS equations (7) that do describe interacting wave trains at arbitrary angles, for which we have worked out the coupling coefficients. This set of coupled NLS equations has exact solutions that qualitatively describe one type of the experiments being conducted by HH.

Is there a critical depth interval at which their exists an intermediate regime where the behavior of bi-periodic traveling waves changes from hexagonal patterns with two crests per period in shallow water to rectangular and/or hexagonal patterns in deep water with one crest per period? We will further explore this question using numerical experiments of Euler's equations, approximate models such as the DS equations (5), and with physical experiments. Again, (5a) and (5b) are not integrable and assume a separation of scales that is not exhibited in these solutions; hence, we will look for more appropriate approximate models for the observations.

The Transverse Envelope: In many of the recent experiments by HH, the shape of the y-envelope of the waves was unexpected. In particular, the wave paddles are programmed to generate plane waves in x with an envelope in y that has the shape of either a Jacobi elliptic sn-function with a modulus larger than 0.9 (an exact solution to NLS (6)), or a cosine modulation (an exact solution to coupled NLS (7)). In almost all cases using the sn-envelope, the time series measured with wave-gages that traverse through the wave patterns give results that cannot be guessed at by looking only at the photographs. For example, in Figure 1 the right-hand photo shows straight crest lines with no modulations. However, the time series shows a large modulation in the envelope. Two types of behavior are observed when using the cosine generated envelopes: In some cases, the time series show envelopes that look like cosines, as expected. In other cases, the envelope "collapses" away from a cosine shape into the sn-shape with a large modulation. A possible explanation for the large modulation is that the waves are undergoing an instability, as a consequence of which they are evolving toward a different exact solution to the approximate (and ultimately full) equations. For example, we are able to find "dn" solutions to (7) that qualitatively describe the observed modulation. We plan to explain these observations with quantitative comparisons with exact solutions to (6) and (7), with predictions of instabilities of these solutions, and with computations of the full equations.

Linear Stability: The stability of deep-water waves has been investigated by many authors in many contexts. The stability of finite amplitude, plane waves to three-dimensional disturbances have been investigated by McLean *et al* (1981), McLean (1982), Zhang & Melville (1987), Kharif & Ramamonjiarisoa (1988, 1990) and others. Dias & Haragus-Courcelle (2000) investigate and review the bifurcation of 2-D waves to 3-D patterns within model equations such as NLS and others. There have also been investigations of stability of

3-D surface patterns. Ioalalen & Kharif (1993) looked at stability of permanent form, doubly periodic waves in deep water to superharmonic perturbations and extended this work to waves in finite depth in Ioualalen *et al* (1996).

Carter (2001) considers the 2-D NLS equations (6), which admit traveling waves of permanent form, given in terms of elliptic functions. Carter shows that every such solution is unstable to a long-wave transverse instability. But (6) is an envelope equation that should describe the waves shown in Figure 1. In that case, Carter's results predict that these waves are unstable, and that the unstable mode leaves the nodal lines straight, but deforms lines perpendicular to these.

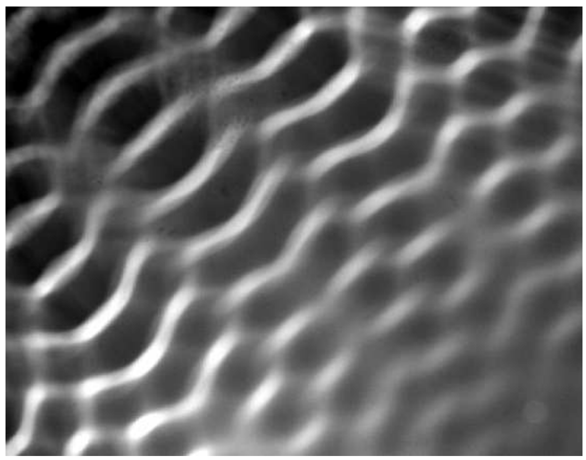

Figure 4: *Overhead photograph of 4 Hz waves in deep water with* $ak = .35$. *(Hammack & Henderson*

Very recent experiments by HH are consistent with this. Figure 4 shows an overhead view of a recent experiment by HH. The wavefield parameters are identical to those shown in the right-hand photo of Figure 1, except that the underlying wave amplitude is 2.3 times larger. Here the crest lines have become curved and the y-envelope displays a large modulation that is more obvious from wave-gage measurements. However, the nodal lines are quite straight. Experiments and calculations are presently planned that will allow for a quantitative test of Carter's (2001) predictions.

D.2.b. Crescents

The experiments of Su (1982) and Su *et al.* (1982) showed moderately large-amplitude waves that developed an instability in which the crests exhibited crescent-type structures. It was reported that these patterns were always forward facing, and this idea then propagated throughout the literature. However, on closer examination, some of the photos published in the papers show that crescents can in fact face either forward or backward. In all cases, Su's papers show that the crescents on the forward face of the waves are more pronounced. They have also been seen in wind-wave tanks by Caulliz *et al.* (1999). Hammack & Henderson propose herein to use the experimental facility to try to answer the question: Do the crescent structures occur on both the forward and backward face of the waves, and under what conditions are the forward-facing crescents dominant?

The experiments of Su *et al.* have been modeled by Shrira, Badulin, & Kharif (BKS) (1996). This model has the characteristic that the crescents are symmetric – they may occur both on the forward and backward faces of the waves because the patterns are transformed to themselves under time reversal composed with a phase shift. Annenkov & Shrira (2000) re-examined this model to look for non-stationary, short-lived patterns with the crescent geometry using an extensive numerical study of possible evolution scenarios. They were able to find, numerically, the phase asymmetry without the requirement of non-conservative effects, although weak dissipation significantly enhanced the asymmetry of the patterns. Craig (2000) showed that the BKS model describes the Stokes wave train and its loss of stability at moderate amplitudes as a Hamiltonian saddle-node bifurcation, which corresponds to the formation of a stable three-dimensional wave pattern that exhibits asymmetric crescent-shaped elements. He introduced and analyzed a more realistic four degree of freedom Hamiltonian model of water waves that has two principal five-wave interactions. It is more complicated and is not completely integrable; nonetheless this model has traveling wave solutions with similar crescent-shaped elements, and others with the hexagonal features of the BKS model. The crescents of this model also may occur on either the forward or backward face of the wave. Herein, Craig proposes to investigate these models further to account for the pronounced forward orientation by adding long-wave short-wave interactions. Corrections of the model to account for the pronounced forward orientation should also be guided by experimental results.

D.2.c. N-Phase ($N > 2$) Riemann theta function solutions

If the number of phases $N > 2$, the resulting KP solution is typically not bi-periodic. Furthermore, there is no physical reason to restrict the analysis to bi-periodic waves. On the other hand, there are no stability results for these solutions. It is possible that only a restricted class of these solutions is stable. In experiments by Hammack *et al.* (1989, 1995), two-phase solutions were observed. Maybe only the two-phase solutions are stable, or maybe only traveling wave solutions are stable, which include the two-phase solutions. Perhaps all N-phase solutions are stable. These questions need to be answered, both on a mathematical level and on an experimental level. To test the applicability of the stable solutions the exact formulas describing these solutions need to be made effective to a level where the formulas can be made to produce numbers, given certain initial data. This is a nontrivial task, which is nearing completion. Once this is achieved, comparison with numerical solutions of the full equations is possible, as is comparison with laboratory experiments. Deconinck & Segur will do this work.

D.2.d. Operational models and transport theory

Operational models are differential equations which are used to describe the time evolution of average characteristics of a sea-state. They are presently the principal tools for sea-state prediction in weather forecasting bureaus, and in actual practice they are used in conjunction with other atmospheric data to make marine forecasts. Decisions about ship routing, aspects of extreme weather warning systems, *etc* are made from these predictions. There are a variety of operational models available, all having in common that they produce a distribution function $W(x, k, t)$, a function of space, wave-vector and time, which embodies a number of the characteristics of the sea-state, and which solves a transport-like partial differential equation. The distribution function W is closely related to the Wigner transform of the complex wave field describing the free surface evolution. An elegant way to derive operational models is to pose the water wave problem as a Hamiltonian system, perform canonical normal forms transformations of this system up to an appropriate order, and then to derive from this the implied differential equation for the distribution W under the appropriate asymptotic assumptions on the wave fields in question. Roughly speaking, operational models are very successful at predicting values of the average characteristics of the sea-state, under normal conditions, with approximately 80% accuracy. Clearly the forecasting community has an interest in increasing this proportion. As well, there are a number of basic questions that are currently being considered by the community of researchers and users of operational models. An important one has to do with the deviation of the actual surface wave field from its description through averaged characteristics. A reasonable amount of the difference can in fact be attributed to local nonlinear dynamics of individual waves or of wave groups, which are sufficiently steep, or large, or for which the envelope characteristics are too rapidly varying for them to be ascribed to the regime of validity of the model. In our FRG program, numerical simulations by Carter & Nicholls will be planned, which are intended to elucidate focusing phenomena, and the resulting amplitude enhancement and wave packet spreading. This work is related to Bateman, Swan & Taylor (2001).

A second and related question has to do with the fact that the distribution W has been obtained through a process of complex phase averaging, from which information about the wave field is lost; it is possible that in the derivation of an operational model of this form important components of the solution are ignored in averaging out the phases of a surface water wavefield. Craig & Segur intend to consider this situation. A collaboration on suggestions for alternative routes to a wave energy transport model will be part of our FRG effort.

A third and more mathematical question has to do with a rigorous understanding of the asymptotic limit described by transport models, in the general context of surface water waves. This has a parallel in the study of the fluid dynamical limit of the Boltzmann equation, a very active subject over the past decade. Craig & Sulem are intending to take up this possibly difficult analytic question.

D.2.e. An inverse problem in two and three dimensions

Shoals and other topography on the bottom of a body of water affect the character of surface waves in a nontrivial way. The inverse problem is to understand the bottom topography given a surface wave pattern, and possibly its dynamics. Craig, Nicholls and Sulem have developed an approach to this problem using the perturbation theory for the Dirichlet-Neumann operator for the fluid domain. Their approach works for a regime in which the mean depth is greater than the typical surface wavelength, but still influences the waves' velocity fields and dispersion relations. We propose to pursue this approach both analytically and with numerical implementations, and to explore the regime of validity of the method. Several initial trials (unpublished) by Craig and Nicholls are very promising.

D.2.f. Dissipation mechanisms for ocean waves.

One of the difficulties modelers of ocean surface waves encounter is how to account for dissipation. Various ad-hoc models are used to account for wave-breaking (white-capping). We have begun to consider two types of dissipation mechanisms. The first uses a set of equations that couples triad and quartet interactions to allow for a transfer of energy from long to short waves but then does not allow the energy to return because of viscous dissipation at short scales. The second uses coupled NLS equations like (7) to model wave focusing and blow-up in the equations that might model the formation of "microbubbles", which leads to white-capping. These ideas have begun to arise due to discussions in our most recent meetings and are still being formed.

D.3 A timeline for the planned work

The project will span three years. This time period is required to obtain and assimilate results. We will meet every six months to discuss progress and modify the plan as required by the results.

Prehistory

Prior to the initiation of this proposed research, the various members of our group have already begun meeting and have produced preliminary results, some of which we have discussed in the previous sections.

1. July 2000 at Brown University: (Craig, Henderson, Nicholls, Segur).

2. August 2000 at Penn State University: (Carter, Craig, Deconinck, Hammack, Henderson, Sulem).

3. April 2001 at University of Georgia: (Carter, Craig, Deconinck, Segur).

4. June 2001 at the Fields Institute, University of Toronto: (Carter, Craig, Deconinck, Hammack, Henderson, Segur, Sulem).
5. July 2001 at University of Colorado: (Carter, Craig, Hammack, Henderson, Segur).
6. August 2001 at the Newton Institute for Mathematical Sciences, University of Cambridge: (Week 1: Craig, Nicholls, Segur. Week 3: Craig, Henderson).

A timeline for the three years of this proposal is as follows.

Year 1

Experiments: In the 2-D tank, JH&DH will examine the evolution of uniform wavetrains with cnoidal wave and dn-wave modulations corresponding to exact solutions of the NLS equation. These experiments will test the predictions of the stability calculations of Carter (2001), by directly seeding the wave patterns with a prescribed perturbation and then measuring the growth rates and the visual manifestation of the perturbation. They will also provide insight on the effects of viscous damping. They will continue their program of experiments in the 3-D tank, which will include a search for the hexagonal patterns predicted by Bridges *et al.* (2001). They will do quantitative comparisons of patterns that appear in the full equations and in the approximate equations. They will test, quantitatively, as in the 2-D tank, the stability predictions of JC & HS for three-dimensional surface patterns.

Numerics: DN will use spatial data obtained from experiments for input to his full Euler's simulations for direct, quantitative comparison.

Analytics: WC & CS will begin looking analytically at the stability of three-dimensional patterns in the full Euler's equations. JC & HS will provide predictions of growth rates for various instabilities. BD will work on translating the N-phase exact solutions of KP and NLS into physical solutions that can be computed and compared to experimental results. DH and HS will explore the consequences of coupled NLS models, like (7).

Year 2

Experiments: JH & DH will continue compiling data sets that include time series and images of the wave patterns. They will start perusing parameter space. In particular, they will begin looking at the waveforms in different depths. They will begin looking for crescent structures with forward and backward orientations by direct generation at the wave paddle. They will begin parametrizing the paddles to generate unsteady wave patterns.

Numerics: DN will conduct simulations in the full Euler equations. JC & HS will develop codes for approximate models including the DS equations as well as higher-order models. BD will begin the actual computing phase of his work on making accessible the N-phase exact solutions. The group will make comparisons among models of the same phenomenon (the full equations, the DS equations, and the KP equation), as well as with experimental results.

Analytics: HS, BD, & DH will work on new approximate models to describe the complicated wave patterns from observations. WC, DN, & CS will work on the inverse problem for extracting bottom topography given the surface wavefield, and forming predictions that can be tested. They will extend previous work on the BKS model for crescents to account for the observed asymmetries.

Practical Models: BD, HS & JH will begin to compare N-phase KP solutions with measured ocean waves in shallow water, as recorded at the U.S.Army Corps of Engineers facility in Duck. NC. (*cf*. Long & Oltman-Shay, 1991.)

Year 3

Experiments: JH & DH will continue experiments on surface patterns. They will use the two-dimensional tank to test the predictions from the inverse topography problem. They will test ideas concerning the asymmetry in the crescent structures.

Numerics: JC & HS will conduct simulations that compare various approximate models. This work has two parts: (1) they will examine the different approximate models to determine how the qualitative behavior of solutions change as higher order equations are used, and (2) they will use experimental data as initial conditions for the different models to test directly predictions of evolution.

Practical Models: BD, HS & JH will continue comparing KP predictions with measured ocean waves. At this point we will have learned whether there are 3-D wave patterns in deep water that propagate with permanent form for long enough times, over long enough distances, so that they can be used to build a practical model for ocean waves. So, in this year we will begin to determine how our work fits into the broader picture of stochastic, ocean wave models, and begin to build a practical theory for waves in deep water that incorporates our results.

D.4 Plans for disseminating the results.

Our main two methods of disseminating results will be to publish papers in refereed journals and to present them at scientific meetings, including the workshop that we are planning.

Because of our varied backgrounds, the work will be shared with a large community including mathematicians, engineers, oceanographers, and other physical scientists. Hammack & Segur have already worked with practicing coastal engineers at CERC (Army Corp of Engineers, Coastal Engineering Research Center, in Vicksburg Mississippi) and with those at the Army Corp of Engineers installation at Duck, North Carolina. These engineers remain interested in the present work. Any collaboration that arises with them is a way of disseminating the results, since they are the end users of the product.

In addition, we are creating a web page that contains relevant experimental data and numerical simulations. At present, photographs and corresponding time series from the experiments are (somewhat haphazardly) available on `http://www.math.psu.edu/dmh/FRG`, a web site that we have been using to share data among the group. In addition we have begun sharing the data with other investigators, such as Per A. Madsen, Professor of Informatics and Mathematical Modeling, at the Technical University of Denmark. After hearing Henderson's talk at the meeting for "Theoretical Developments on two and three dimensional water waves" at the Newton Institute in Cambridge in August, 2001, he expressed an interest in obtaining the data so that he could simulate the nonlinear surface patterns (private communication). Deconinck is constructing our web site for the general public at `http://www.math.colostate.edu/~deconinc/frg_surfacewaves.html`. This web site will display the data in a more accessible manner than does the present web site.

Deconinck will finish his MAPLE program, which is, in its present state, and will be, in its finished state, part of the update version of MAPLE. This program will allow users to compute with Riemann surfaces and Riemann theta functions using a handful of interactive commands. We will make the results accessible on a popular level during the laboratory tours that are regularly given to students from local math clubs, to home-schoolers, and to other groups. In addition, the wave tanks will be used in hands-on projects by girls during our outreach programs, such as "Math Day", for 9th & 10th grade girls", and "Expanding Your Horizons", for 11th grade girls.

D.5 Results from prior NSF support.

John Carter was supported as an NSF-VIGRE (DMS-9810751) graduate student and also by NSF-DMS-9731097 at the University of Colorado, where he worked with Harvey Segur. During this time he developed numerical codes for integrating the NLS equation (6) and its higher-order generalizations. He will continue exploring these generalizations during the tenure of the proposal to see how their predictions compare with those of (6). His thesis resulting from this support corresponds to reference Num in Section E and dealt with the stability of exact solutions to (6). We have begun discussing his results within the context of the physical experiments and plan to conduct a quantitative comparison between the two.

Walter Craig's present NSF award is DMS-0070218 in the Program in Analysis, entitled *Methods of Hamiltonian Mechanics for Nonlinear Wave Equations* and with award amount $50,000 per year. His research projects under this grant and for the past three year grant period can be roughly divided into the categories (1) KAM theory and analytic methods for Hamiltonian PDE, (2) Euler flows with free surfaces, (3) dispersive smoothing and the propagation of singularities for the Schrödinger equation, and (4) Lyapunov exponents for PDEs with random coefficients. During the above award periods this research resulted in nine papers which appeared in print in refereed journals, one paper which has been submitted and is currently being refereed in such a journal, and three articles which appear in volumes of conference proceedings. In addition his monograph on small divisor problems has appeared in "Panoramas et Synthèse". Bibliographic references to this work correspond to references Num-Num listed in Section E.

Bernard Deconinck worked with Harvey Segur as an NSF-supported graduate student (NSF-DMS-9731097) at the University of Colorado. There he developed a constructive method to solve the initial-value problem for multiphase solutions of the KP equation. Publications from this support include references Num-Num in Section E.

Deconinck's construction makes use of earlier work by Krichever (1977). However, Krichever's ingenious method to construct these solutions from certain data associated with Riemann surfaces did not allow for the computation of the multiphase solutions of the KP equation. In order to be able to use these solutions as practical tools for comparison with experiments and numerical simulations, BD started a long-term project to make Krichever's method effective. The first step in this project was a collaboration with Mark van Hoeij (FSU), which started while BD was an NSF supported fellow at MSRI during 1998-1999 (DMS-9701755). The outcome of this collaboration was not only the first (and hardest) step in the effectivisation of Krichever's procedure; it also provided several black-box programs (now included in Maple6) for symbolic computation with Riemann surfaces. For the first time, it provided teachers of Riemann surface theory with nontrivial concrete examples of concepts that until now were part of abstract mathematics. These black box programs are discussed in reference Num in section E. They are also available from `http://www.math.fsu.edu/ hoeij/periodmatrix/`.

After MSRI, BD was an NSF postdoctoral fellow at the University of Washington (DMS-0071568). There, he began the implementation of Riemann theta functions at the level of black box programs. This step has now been completed and is listed as reference Num in Section E. BD is now working on the next stage of this long-term project, which will eventually reduce the computation of the multiphase solutions of the KP equation (and other integrable equations) to a black-box program, making these solutions available to whomever wants to use them, without requiring knowledge of the mathematics behind them.

Prior NSF funding for Diane Henderson (DMS-9257456) and Joe Hammack & Diane Henderson (DMS-9972210) was used in part to purchase and fabricate the experimental facility housing the three-dimensional wave basin described in Section H. Using support from DMS-9972210 JH, DH and collaborators investigated two- and three-dimensional waves. The three-dimensional facility will be used further in the present study to examine predicted growth rates for instability of exact solutions of NLS. The previous results are described in references Num-Num in Section E. Additional relevant results from DMS-9257456 include references Num-Num in Section E.

Dave Nicholls had an NSF graduate fellowship when he did his graduate work at Brown University with Walter Craig. Results from this period include references Num-Num in Section E. He recently finished a Dunham Jackson Assistant Professor at the University of Minnesota with NSF summer support DMS-0072462. Results from this period include references Num-Num in Section E. He is now an Assistant Professor at Notre Dame University.

With previous NSF support (DMS-9304390 and 9731097), Harvey Segur worked (in part) on various aspects of the dynamics and mathematics relevant to shallow and deep water waves. Results relevant to this proposal are in references Num-Num in Section E.

Modes of Collaboration and Training

D.6 Training of students and post-doctoral fellows.

As mentioned earlier, our research group has been meeting on and off for a year. During that time, we were already training students and post-doctoral fellows. Specifically:

1. Bernard Deconinck was an NSF Postdoctoral Fellow at the University of Washington. He is now Assistant Professor at Colorado State University.

2. David Nicholls held a named Assistant Professorship at the University of Minnesota. He is now Assistant Professor at Notre Dame University.

3. John Carter was a graduate student at University of Colorado. He will receive his PhD in December, 2001. He is now Assistant Professor at Seattle University.

Future students and postdoctoral researchers involved in this project will have the unusual opportunity to work on a problem from the analytic, numerical and experimental sides. Actually, they will not be required to conduct all three aspects of the problem; however, because of the "focused" aspect of the project, the different brands of students and faculty (experimental, numerical, analytic, and cross-overs) will be meeting twice a year and will be talking to each other and working together. During our six meetings over the last year, we have learned of the importance of talking face to face. The success of one part of the project depends symbiotically on the other parts of the project. The interaction during the group meetings leads to an in-depth understanding of each others problems and pitfalls at a much more fundamental level than would be possible without them. The students and postdoctoral researchers are part of these interactions and have the opportunity to see and understand the project from a global perspective and are forced to encounter the problems associated with the different approaches to research. It is our hope that this global view will have a significant impact in producing an interdisciplinary approach to science in their future careers.

We are requesting support for one graduate student and one postdoctoral researcher. The student will be at Penn State working with JH & DH. An additional graduate student will be supported at CU with NSF-VIGRE monies and will work with HS. A likely candidate for the Penn State student is a new graduate student, Tiffany Pritt, who was funded during summer, 2001, with the NSF grant of JH & DH. She has begun a research project on a new wavemaker problem that incorporates experiments, analysis and numerics. This project has a direct bearing on the experiments proposed herein. She intends to finish this project and pass 2 out of 3 qualifying exams by summer 2002, in time to begin work on the research proposed herein. The students will participate in the bi-annual group meetings and additionally, will have regular medium term visits to work with CS & WC at Toronto/McMaster. Penn State also has support available for undergraduates through their NSF-VIGRE program. We plan to use this resource to incorporate undergrads into the project.

The post-doc will begin her/his work at Penn State, where (s)he will observe the physical experiment, the measurements, and the general overview of the research problem. There, (s)he will conduct experiments and analyze data. During the second year, (s)he will have the option to be located at the University of Colorado where (s)he will work further on the problem from the point of view of the approximate models. In either case, direct comparison with experimental work at Penn State and results from approximate models will be done during this time. Finally, (s)he will have the option to spend the third year at Toronto/McMaster, working on the problem from the point of view of analysis of the full equations. Thus, the post-doc may work on all aspects of the same problem with more than one person. (S)he will have the opportunity to help build a theory for the waves (s)he personally observes. One candidate for this post-doc position is Katherine Socha, who is graduating from the University of Texas under Jerry Bona. She is applying for an NSF postdoctoral fellowship with Henderson as a sponsor, so it is possible that her support may not be required herein. An additional post-doc may work on the project with support from Penn State's NSF-VIGRE program.

D.7 A description of the planned workshops and list of tentative participants

Four members of our group (Craig, Henderson, Nicholls and Segur) attended a workshop on "Three-dimensional aspects of nonlinear water waves" during 12-31 August 2001, organized by S.E. Belcher (Reading), T.J. Bridges (Surrey), and S.G. Sajjadi (Salford) at the Isaac Newton Institute in Cambridge, England. This comprehensive meeting included a variety of investigators, ranging in scope from mathematical analysts that prove existence theorems to scientists that work with satellite radar returns. This meeting ended with a list of open questions with regard to understanding, modeling, and observing 3-D, ocean surface waves. From this meeting, our group obtained a firm idea of how our work fits into the bigger picture of operational ocean wave modeling. We hope that much progress from us and others will be made on these open questions during the next three years, and will plan a similar workshop for the summer, 2004. We expect additional support for it from the Canada Research Chairs Program of Walter Craig. The speakers will include analysts, experimentalists, numericalists, and observationalists. It will include academicians in mathematics, oceanography, and coastal engineering. It will include investigators who collect and observe field data both in coastal regions and in the open ocean, as well as practicing coastal engineers from two US Army Corp installations.

A tentative list (in addition to members of this group), is S. Badulin, M. Banner, T. Beale, J. Bona, H. Branger, T. Bridges, R. Camassa, G. Caulliez, D. Chalikov, M. Chen, R. Dalrymple, F. Dias, M. Donelan, K. Dysthe, S. Elgar,R. Guza, K. Hasselmann, S. Hasselmann, T. Herbers, D. Holm, R. Holman, N. Huang, M. Ioualalen, P. Jennsen C. Kharif, O. Kimmoun, C. Long, M. Longuet-Higgins, P. Madsen, C. Mei, K. Melville, P. Milewski, J. Oltman-Shay, A. Osborne, D. Peregrine, A. Roberts, N. Scheffner, V. Shrira, M-Y. Su, K. Trulsen,

V. Zakharov, J. Zhang.

D.8 Management plan

The group effort will be coordinated around the semi-annual group meetings, which will be hosted by one of the PIs. The host will be responsible for arrangements. Before the meeting, the host will provide a summary that outlines where the project stands at that time. (S)he will make a tentative schedule of lectures, discussions, and coordinated work time. (S)he will give a summing-up at the end of the group meeting. Afterward, the host will write up a summary of what happened. The tentative schedule of group meetings is:

1. July 2002: WC, McMaster U. or Fields Institute in Toronto
2. January 2003: HS, U of Colorado
3. July 2003: JC, U of Seattle
4. January 2004: BD, Colorado State U.
5. July 2004: general workshop
6. Spring 2005: DH, Penn State U

Decisions regarding the budgeting of monies will be made by consensus if at all possible. Since our areas of expertise are so varied, scientific decisions have to be made by the experts in the various areas. Henderson will be responsible for interacting with NSF and for ensuring that the various deadlines and commitments are met.