# Annual Report: NSF–FRG: DMS–**0351466**

John Carter, Walter Craig, Bernard Deconinck, Joe Hammack,
Diane Henderson, Dave Nicholls, Harvey Segur, Catherine Sulem

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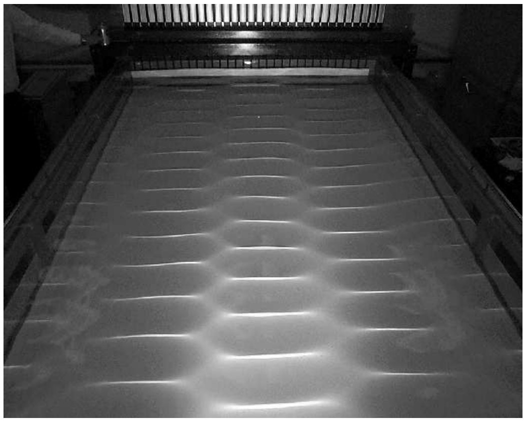
## 1 Introduction

The objective of our FRG is to understand the dynamics of surface water waves, where the waves are three–dimensional, have finite amplitudes, and propagate in water of arbitrary depth. Our plan is that with a deeper understanding of the dynamics, we can construct appropriate building blocks of a practical theory of water waves, without the limitations inherent in other theories. Members of our group use a variety of methods and tools in this study, including laboratory experiments, numerical computation, other approximate methods, and mathematical analysis. A strength of our group is our ability to compare results from different approaches to the same problem. In organizing this report, it is convenient to distinguish between waves in deep water (where the bottom topography plays no role in the wave dynamics), and waves in water of finite depth (where the waves feel the effects of the bottom topography).

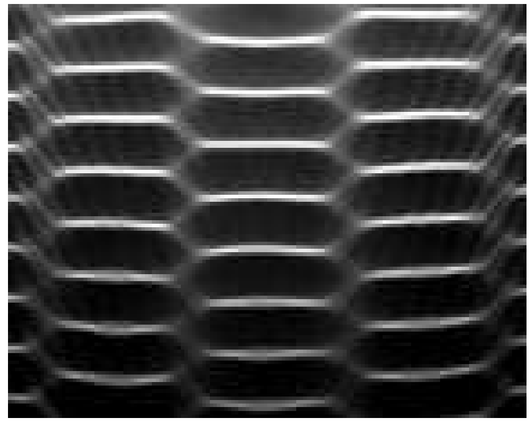
## 2 Waves in Deep Water

A fundamental concept in the theory of inviscid water waves is that a nearly uniform train of plane waves of finite amplitude is unstable in deep water. This instability was discovered nearly forty years ago in groundbreaking work by Lighthill (1965), Benjamin & Feir (1967), Benney & Newell (1967), Ostrovsky (1967), Whitham (1967), and Zakharov (1967, 1968). In water waves, the mechanism is often called a "Benjamin–Feir instability". Essentially the same instability occurs in optics and in plasmas, where it is also known as a "modulational instability". In any of these settings, one shows that a "carrier wave" (i.e., a uniform train of 2D waves of finite amplitude) is unstable to arbitrarily small perturbations of other waves with nearly the same frequency, propagating in nearly the same direction. Whitham (1967) showed that for water waves, the instability occurs only if the water is deep enough ($kh > 1.36$, where $k$ is the wavenumber of the carrier wave, and $h$ is the water depth). Therefore, this instability is one of the fundamental differences between wave propagation in deep and shallow water.

Our experimental work on waves in deep water has forced us to re–examine the practical significance of this instability. In experimental work reported last year, Hammack & Henderson (2003) exhibited wave patterns of 3D surface waves that propagate in deep water with little evidence of a Benjamin–Feir (or BF) instability. Figure 1 shows photographs of two such trains of 3D waves, each periodic in two horizontal directions, each propagating in deep water with no obvious instability. Other photographs and movies of wave patterns like these can be found at the central website of our FRG:



(a) Oblique view



(b) Overhead view

Figure 1: Photographs of typical bi–periodic patterns of surface waves in deep water with different aspect ratios and amplitudes. (a) oblique view showing the tank and wavemakers, (b) overhead view.

`http://www.amath.washington.edu/~bernard/frg_surfacewaves.html`

under "Experiments," and then "Waves with 2D patterns in deep water." One sees there several examples of wave patterns that propagate in deep water with little evidence of instability. These experiments suggest that the BF instability may not be as ubiquitous as is commonly believed. If correct, this possibility would profoundly change our picture of waves in deep water: Perhaps stable wave patterns exist in deep water. Much of the work of our FRG on waves in deep water has been aimed at exploring this possibility, from different viewpoints, using different tools.

## 2.1 Experimental Work

In a paper consisting primarily of experimental observations, Hammack, Henderson & Segur (2004) document the behavior of these (seemingly stable) 3D wave patterns as they propagate in deep water. This paper organizes the observations by identifying a set of "features" that regularly appear in these wave patterns. Additionally, the paper provides two models that may be used as starting points for explaining the features. The paper's main contribution is that the list of features provides a standard by which to judge any subsequent theory of these waves. This paper is now under review. To obtain a preprint of the manuscript, go to:

`http://www.amath.washington.edu/~bernard/frg_surfacewaves.html`

click on "Experiments," then on "Waves with 2D patterns...," and then on "Persistent Wave Patterns."

## 2.2 Instability in the NLS Model

Zakharov (1968) derived the nonlinear Schrödinger (NLS) equation in 2D,

$$i\partial_t A + \alpha\partial_x^2 A + \beta\partial_y^2 A + \xi|A|^2 A = 0, \tag{1}$$

to describe approximately the slow evolution of a train of nearly monochromatic waves, with small but finite amplitude. In this model, $\{\alpha, \beta, \xi\}$ are real numbers defined by the experiment in question. For gravity-induced waves in deep water $\{\alpha < 0, \beta > 0, \xi < 0\}$. In any such setting, the solution $A(x, y, t)$ represents the complex envelope of the oscillatory waves, so a spatially uniform train of oscillatory waves corresponds to $A = A(t)$, with no spatial dependence in the envelope. Demonstrating the BF instability of a uniform wavetrain in this model amounts to showing that $A = A(t)$ is unstable to small perturbations in (1).

The doubly periodic wave patterns shown above in Figure 1 also can be represented within the NLS model. One can verify directly that

$$A = \sqrt{-\frac{2\beta}{\xi}}\ (ck)\ \mathrm{sn}(c\ y, k) e^{-i\beta c^2(1+k^2)t} \tag{2}$$

solves NLS, where $\mathrm{sn}(z, k)$ is a Jacobi elliptic function with elliptic modulus $k$. Here $A$ represents the envelope of a carrier wave (with oscillations in the $x$–direction), and $A$ itself oscillates in the $y$–direction, so the wave pattern has the doubly periodic spatial structure seen in Figure 1. Figure 2, taken from Hammack, Henderson & Segur (2004), shows an overhead image and a time series of one of the doubly periodic wave patterns observed experimentally, and the corresponding two images of the NLS solution in (2). The qualitative correspondence of the two images is evident.

Thus, NLS is a plausible approximate model of the evolution of these doubly periodic wave patterns. Are the wave patterns stable, according to this model? The solution in (2) occurs if $\beta\xi < 0$ in NLS. Other elliptic function solutions occur if $\beta\xi > 0$. Carter & Segur (2003, supported by this grant) showed that for either choice of these signs, any elliptic function solution of NLS like that in (2) is linearly *unstable* to small transverse perturbations. The growth rates of these instabilities are comparable to those of the BF instability of a uniform wave train. In this sense, one can view the instability of the solution in (2) as a generalization of the usual BF instability.

Thus, according to the NLS model, a uniform train of oscillatory plane waves is unstable, a train of 2D doubly periodic waves is also unstable, and their growth rates are comparable. However, the stability of the wave patterns shown in Figure 1 (and of a uniform train of plane waves) is more complicated than this, as we discuss next.

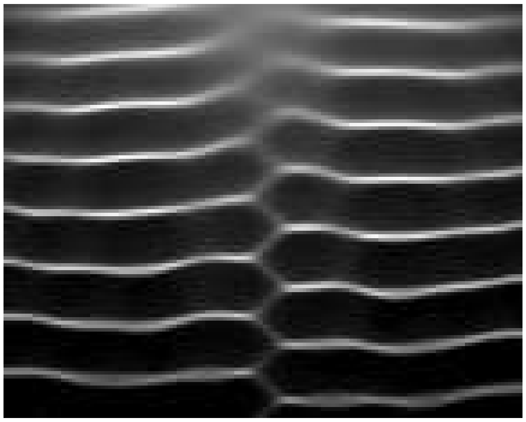
## 2.3 Stability of a Uniform Train of Plane Waves in a Damped NLS Model

As discussed above, the NLS model arises as an approximate model of wave propagation in many physical contexts. In almost every physical context, some form of wave damping occurs naturally, but this effect is omitted from the NLS model. Several papers have corrected this deficiency of the NLS model by including a damping term,

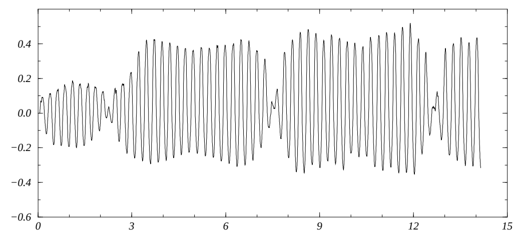
$$i\partial_t A + \alpha\partial_x^2 A + \beta\partial_y^2 A + \xi|A|^2 A + i\delta A = 0, \tag{3}$$

where $\delta \geq 0$. Miles (1967) derived analytic formulas for $\delta$, based on specific kinds of damping that occur for waves in deep water.

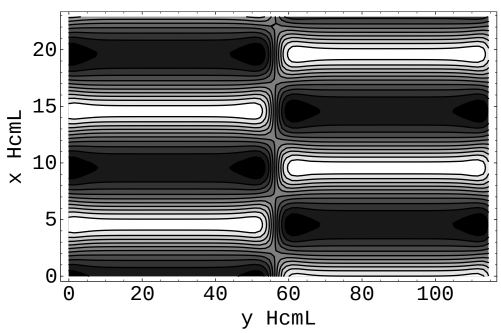
In a recent paper supported by this grant, Segur, Henderson, Carter, Hammack, Li, Pheiff, & Socha (2004) used (3) to reconsider the BF instability of a uniform train of plane (i.e., long–crested) waves. Our main theoretical results follow.



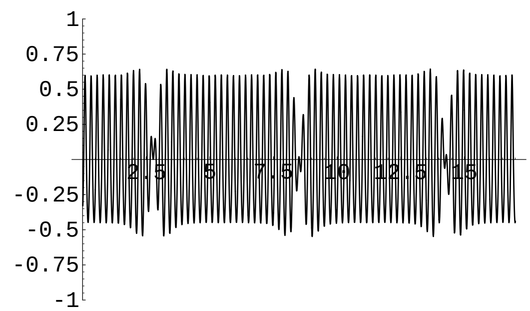
(a) Experimental Contour




(b) Experimental Time Series




(c) Theoretical Contour




(d) Theoretical Time Series

Figure 2: Waves using (2) as wavemaker forcing and theoretical comparison: (a) experimental contour map obtained from overhead photograph, (b) experimental time series measurement taken by traversing a wave gage parallel to the wavemaker array, (c) theoretical contour map of the third order solution to the water wave problem based on (2) and the experimental parameters of (a)–(b), (d) theoretical time series obtained by making a cut across (c).

**Theorem 1 (BF instability without damping).** *The spatially constant solution of NLS (equivalently, of* (3) *with* $\delta = 0$*) is*

$$A = M \exp\left\{ i\xi |M|^2 t \right\},$$

*where* $M$ *is an arbitrary complex constant. For any choice of signs in NLS* ***except*** $\{\alpha\beta > 0, \alpha\xi < 0\}$*, such a solution is unstable to small perturbations. Specifically for waves in deep water, where* $\alpha\beta < 0$*, this solution is unstable.*

**Theorem 2 (Stability with damping).** *The spatially constant solution of* (3) *with* $\delta > 0$ *is*

$$A = M \exp\left\{ i\xi |M|^2 \left( \frac{1 - e^{-2\delta t}}{2\delta} \right) \right\}, \tag{4}$$

*where* $M$ *is an arbitrary complex constant. For any* $\delta > 0$ *and any choice of signs of* $\{\alpha, \beta, \xi\}$*, this solution is both linearly and nonlinearly* ***stable*** *to small perturbations.*

Thus, it follows from (3) that any amount of damping (i.e., any $\delta > 0$ in (3)) stabilizes the BF instability. This result surprised us, and it has been controversial. Some have questioned the validity of (3) as a model of waves in deep water. To answer this question, we tested the model with laboratory experiments. Figure 3a shows a modulated train of plane waves, as it propagates past a fixed location in a wave tank. Figures 3c and 3e show the same wavetrain at two locations further down the tank. It is clear from these data that the wavetrain decays in amplitude and evolves in shape as it propagates down the tank. Even so, (3) describes this entire process with good accuracy. Using the data from Figure 3a as initial data for (3), we predicted the values of 8 different measures of this wave train, each at 11 measuring stations further down the tank, including those shown in Figures 3c, 3e. The theory predicts these data with remarkable accuracy, and it shows that there is no BF instability for $\delta > 0$.

We find that (3) works very well for waves of small–to–moderate amplitude in deep water; i.e., in the situation that Benjamin & Feir studied in their 1967 paper. We also find that (3) is inadequate for large amplitude waves, which exhibit downshifting during their evolution. We plan to analyze large amplitude waves in a separate paper. This paper is now under review. For a preprint, go to:

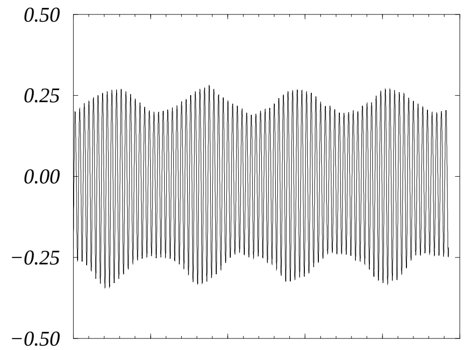
`http://www.amath.washington.edu/~bernard/frg_surfacewaves.html`

click on "Experiments," then "Waves with 2D patterns. . . ," and then "Stabilizing the Benjamin–Feir instability."

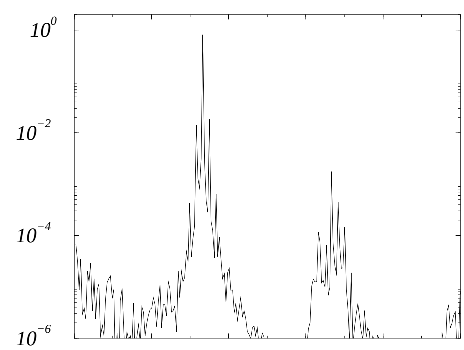
## 2.4 Work in Progress: Stability of Doubly Periodic Wave Patterns in Deep Water

Because of the striking experimental demonstration of doubly periodic wave patterns that seem to persist in deep water, described above, our FRG has devoted considerable effort to determining the stability of these wave patterns. Two of the papers described above use different mathematical models to study the stability of waves in deep water. The two (ongoing) projects described next use two other mathematical models to study the stability of doubly periodic wave patterns in deep water.

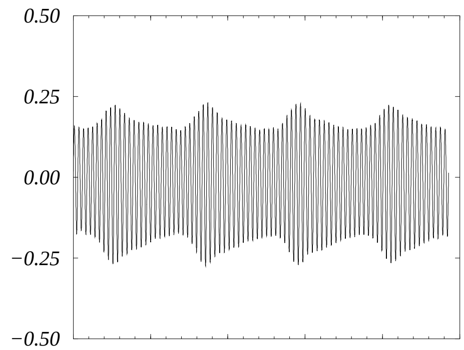
Thus, our FRG is studying one general question (stability of doubly periodic waves in deep water) in terms of several mathematical models. We regard this ability to examine the



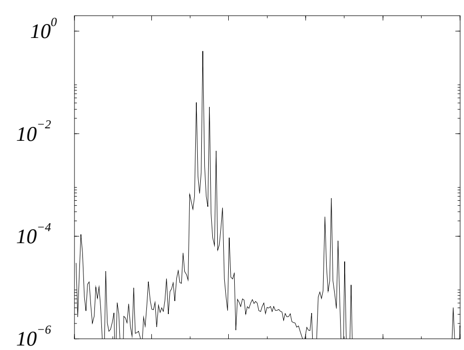

(a) Time Series ($x = 50$)



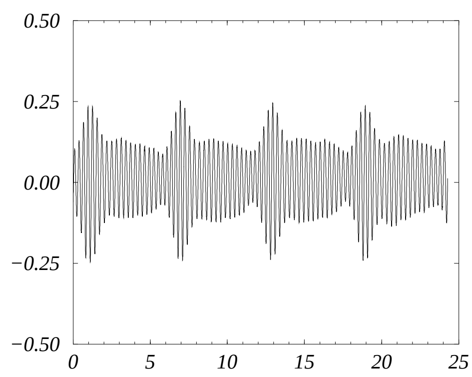

(b) Fourier Transform ($x = 50$)



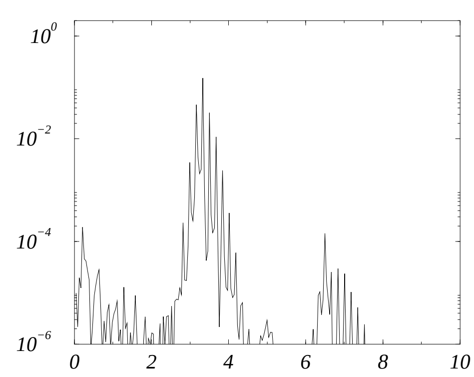

(c) Time Series ($x = 300$)




(d) Fourier Transform ($x = 300$)




(e) Time Series ($x = 600$)




(f) Fourier Transform ($x = 600$)

Figure 3: Time series (a), (c), (e) and Fourier Transforms (b), (d), (f) for a modulated wavetrain in the wave channel measured at three locations downstream of the wavemaker.

same question from several viewpoints as a strength of our FRG. The different mathematical models incorporate different physical effects, so comparing results from different mathematical models suggests how various physical effects might affect the stability/instability of these waves. In addition, our FRG generates experimental data, so we can compare mathematical models to each other and also with experimental data.

#### 2.4.1 Stability of Doubly Periodic Waves: Euler's Equations

In earlier papers, Craig & Nicholls (2000, 2002) proved that the full equations of inviscid water waves (which we call "Euler's equations" herein) admit traveling wave solutions with doubly periodic surface patterns, like those shown in Figures 1 and 2. Their results hold for water of any (uniform) depth; the results for deep water discussed here are a special case.

Euler's equations provide another mathematical model of these waves–one can think of NLS as an approximation to Euler's equations. The question is: Are these doubly periodic traveling wave solutions of Euler's equations stable to small perturbations, according to Euler's equations?

W. Craig, D. Nicholls and C. Sulem are developing a theory for the linearized stability of the traveling wave solutions of Euler's equations, found by Craig & Nicholls (2000, 2002). Their theory has the structure of a Bloch theory (from condensed matter physics), in parallel with the band structure of conduction zones in the space of quasi–momenta. Their work will extend the three–dimensional stability calculations of McLean *et al.* (1981) and Ioulalen, *et al.,* (1993,2002).

Craig, Nicholls & Sulem are also developing a numerical code designed to handle large–amplitude waves with periodic patterns. Numerical results from this code will be compared to earlier numerical results by Ioulalen, *et al.,* (1993, 2002), and to experimental data.

#### 2.4.2 Stability of Doubly Periodic Waves in the Presence of Damping

Generalizing the work of Segur *et al.* (2004, cited above) on stability of plane waves in the presence of damping, M. Bleymaier and H. Segur have begun work on the stability of a train of doubly periodic waves, like those shown in Figure 1, in the presence of damping. As mentioned above, traveling wave solutions of Euler's equations provide one model of these doubly periodic waves; (2) provides another model, within the NLS equation. A third approximate model, based on two coupled NLS equations, is described by Hammack, Henderson & Segur (2004).

Bleymaier & Segur have added a small amount of damping to each of the coupled NLS equations, comparable to what is done in (3) for NLS. They have found the family of solutions of the damped, coupled equations that describes the doubly periodic wave patterns, generalizing (4). The next step is to analyze the linearized stability of these solutions within the model of the damped, coupled NLS equations. The results of these calculations can then be compared directly with the experimental observations described by Hammack, Henderson & Segur (2004). This project is in its early stages.

### 2.5 Work in Progress: Other Stability Results

Stability plays a central role in the current work of our FRG, and we have developed other results about stability in more general contexts.

### 2.5.1 Elliptic Function Solutions of NLS

The results of Carter & Segur (2003) establish the instability for any elliptic–function solution of NLS like that in (2). These solutions are periodic, with a period that depends on the elliptic modulus, $k$. Solitary waves arise as limits of the periodic solutions, when $k \to 1$. However, this limit is delicate, and it is not easy to infer results about solitary waves from the periodic case. J. Carter and B. Deconinck have now worked out the solitary wave situation directly.

### 2.5.2 Floquet Theory for Nonlinear Wave Equations

B. Deconinck and J.N. Kutz have been developing a general method for the linear stability analysis of stationary periodic and solitary waves solutions of nonlinear wave equations. The method is not specific to NLS, but it was first devised in this context, and it was applied there first. Like the work of Craig, Nicholls & Sulem (cited above), their theory builds in Floquet theory (which is closely related to Bloch theory), and seems to be very fast computationally.

### 2.5.3 Damping of Other Nonlinear Wave Equations

As discussed above, Segur *et al.* (2004) have shown that damping stabilizes the Benjamin–Feir instability of a uniform train of nearly monochromatic plane waves in deep water. In the same spirit, N. Canney and J. Carter have added damping to higher–order models of waves in deep water, models derived by Dysthe (1979) and Trulsen & Dysthe (1996). They seek to answer the same question: Does damping stabilize the instability of nearly monochromatic plane waves in deep water, according to these models? This is another example of using several mathematical models to answer the same question, in order to learn how robust the answer is. This project is in its early stages.

# 3 Waves in Water of Finite Depth

## 3.1 Problem Formulation and Approximate Models

The landmark paper by Zakharov (1968) contained several important results, including an elegant Hamiltonian formulation of Euler's equations of inviscid water waves. In a recent paper, W. Craig, P. Guyenne, D. Nicholls, and C. Sulem (2003, supported by this grant) used this formulation to develop a long–wave perturbation theory, using Birkhoff's formal procedure of canonical transformations of the Hamiltonian system. As one would expect, the procedure recovers the standard approximate models for long waves in shallow water: the Korteweg–de Vries, Boussinesq, Kadomtsev–Petviashvili and Davey–Stewartson equations. The paper's new contribution is to generalize these known equations to include the effects of a non–flat bottom. The analysis includes the effects of bottom topography with both single and multiple scales, and it generalizes earlier work by Rosales & Papanicolaou (1983). This work is now under review.

In related work, also supported by this grant, W. Craig, P. Guyenne and H. Kalish (2004) have used similar ideas to study internal waves of large amplitude and long wavelength.

D. Nicholls and F. Reitich (2004) recently submitted a paper on the analyticity of traveling water waves. They prove analyticity with respect to both the perturbation parameter and the spatial variables. It generalizes earlier work by Reeder and Shinbrot (1981) to include traveling patterns that are not necessarily short–crested. For a preprint of this work, go to:

`http://www.amath.washington.edu/~bernard/frg_surfacewaves.html`

click on "Publications," and then on "On analyticity of traveling water waves."

### 3.2 Accurate Numerical Schemes

Zakharov's Hamiltonian formulation of Euler's equations can also be incorporated into numerical schemes, where the inherent symplectic structure can be used to increase the accuracy of the computation. P. Guyenne & D. Nicholls (2004) used this idea to study numerically the motion of solitary waves over variable topography. For a preprint of this work, go to:

`http://www.amath.washington.edu/~bernard/frg_surfacewaves.html`

click on "Publications," then on "Numerical simulation of solitary wave on plane slopes."

### 3.3 Completely Integrable Models of Water Waves

The discovery and development of partial differential equations that are completely integrable has included several equations that also arise as approximate models for water waves, including 1D NLS (1) in deep water, and the famous Korteweg–de Vries (KdV) and Kadomtsev–Petviashvili (KP) equations in shallow water. These special equations exhibit enormous mathematical structure, including huge families of exact solutions. The families can be described most completely in terms of Riemann theta functions, but this correspondence has had limited practical value because the theory of Riemann theta functions is not yet effective in a practical sense.

Motivated by the need to use these special equations as models of water waves and other physical phenomena, B. Deconinck and his collaborators have been developing the missing ingredients in the theory of Riemann theta functions. Deconinck, Heil, Bobenko, van Hoeij & Schmies (2004, supported by this grant) present an algorithm for the uniform approximation of Riemann theta functions of arbitrary genus, providing an efficient way to compute these functions. If one thinks of KP as a model of nonlinear, 3D waves in shallow water, then a KP solution of genus $N$ represents a periodic or quasi–periodic wave pattern with $N$ phases, each phase corresponding to a periodic wave, traveling in its own direction with its own frequency, and with the waves interacting nonlinearly with each other in 3D. It is a remarkable feature of integrable models like KP that this much information can be encoded in a solution that one writes down explicitly.

In the next step needed to make these solutions effective, B. Deconinck and M. Patterson are finishing a practical method to implement the "Abel transform," which is needed to calculate Riemann theta function solutions of arbitrary genus. Once finished, this machinery can be used to calculate $N$–phase quasi–periodic solutions of models of waves in deep water (like NLS), and other solutions of models of waves in shallow water (like KP, KdV or Boussinesq). This work will be part of M. Patterson's PhD thesis.

### 3.4 Work in Progress: Interacting Solitary Waves in Shallow Water

Part of the rich mathematical structure of the KdV equation, which models waves propagating in one direction in shallow water, is exhibited by the overtaking collisions of its soliton solutions. See Cooker, Weidman & Bale (1997) for a review of previous investigations of collisions of solitary waves in shallow water. As part of this FRG, we are examining collisions of solitary waves using experiments, Euler's equations, and coupled KdV equations. J. Hammack, D. Henderson, M. Yi (a high-school student) and H. Tu (an REU student), conducted experiments in the laboratory's wave channel. In these experiments, precise KdV solitons were generated to investigate both overtaking and head–on collisions. The experiments were designed to provide precise spatial profiles of the water surface as a function of time. The experimental results are now being used to compare with three different mathematical models of waves in shallow water.

P. Guyenne, in collaboration with W. Craig, is computing time–evolving solutions from Euler's equations, using the experimental data as his initial data. D. Wright, in collaboration with W. Craig and C.E. Wayne, is computing time–evolving solutions of coupled KdV equations using the experimental data as initial data. Wright derived and analyzed these equations as part of his Ph.D thesis (2004). M. Chen at Purdue is investigating soliton collisions using a Boussinesq model. Comparisons of the theories with the data are now under way. Preliminary results are available at:

`http://www.amath.washington.edu/~bernard/frg_surfacewaves.html`,

click on "Experiments," and then on "Collisions of Solitary Waves."

### 3.5 Work in Progress: Wave Measurements in Shallow Water

An interesting feature of experimental data on waves in shallow water is that waves are often measured differently in laboratory and field experiments. In laboratory experiments, the location of the water surface is often measured using a capacitance gage at the water surface; in field data, the location of the surface is often inferred from pressure measurements, taken at the ocean floor. In converting pressure measurements to surface elevations, the usual assumption of simple hydrostatic pressure is inadequate for steep, nonlinear waves. B. Deconinck, J. Hammack and M. Patterson are beginning to use nonlinear wave theory to obtain more accurate estimates of surface elevations from pressure measurements. They will start by using pressure gage data obtained in the lab, for which explicit surface elevation data also exist. Once they can convert pressure measurements into surface elevations correctly in the lab, then they can compare pressure–gage data obtained at the Field Research Facility in Duck, N.C. with predictions from KP theory. This work has just begun.

## 4 Organization and Management of the FRG

### 4.1 Personnel

As the work cited in this report shows, contributors to this effort include the eight people directly involved in the FRG, plus other collaborators. In addition to working scientists, our collaborators at this time include a high school student (*Penn State*: M. Yi), undergraduate students (*Penn State*: D. Pheiff, H. Tu; *Notre Dame*: B. Dolan, *Seattle U.*: N. Canney, E. Hunt), graduate students (*Colorado*: M. Bleymaier; *Notre Dame*: M. Taber;

*U. of Washington*: M. Patterson), and postdocs (*McMaster*: P. Guyenne, D. Wright). In addition, R. Thelwell, who has just completed his PhD at Colorado State, will begin work as a postdoc supported by this grant. His base will be at the U. of Washington, but he plans to travel to the other sites of our FRG in his work.

### 4.2 Scientific Meetings

Our Focused Research Group has eight principal investigators, located at seven institutions in five states and provinces. Communication is important for any functioni ng research group, but it is critical for a far–flung group like ours. Telephones and email solve some communication problems, but we also need timely face–to–face communication to make real scientific progress.

In the last year, we have organized four scientific meetings involving three or more of us. The times, locations and participants (including students and postdocs, when appropriate) for these meetings are as follows:

- *June 20–23, 2003, Colorado State U., Fort Collins, CO* – B. Deconinck, J. Hammack, H. Segur
- *November 14–18, 2003, the Fields Institute, Toronto, Canada* – J. Carter, W. Craig, B. Deconinck, P. Guyenne, D. Henderson, H. Segur, C. Sulem, D. Wright
- *Jan. 29–Feb. 1, 2004, Penn State, State College, PA* – M. Bleymaier, J. Hammack, D. Henderson, H. Segur
- *March 15, 2004, Seattle U., Seattle, WA* – N. Canney, J. Carter, B. Deconinck, D. Henderson, M. Patterson, H. Segur

In addition, we have organized an international Workshop on Free–Surface Water Waves, to be held at the Fields Institute during June 14–18, 2004. All of us plan to attend that meeting.

### 4.3 Dissemination of Results

The most reliable way to disseminate scientific results is to publish them in well–established scientific journals. In the last year, members of our FRG have published four papers, and have submitted five more. These are listed below, in Section D.

In addition, we maintain a central website for our FRG, at:

`http://www.amath.washington.edu/~bernard/frg_surfacewaves.html.`

This site has links to the websites of individual investigators, and it serves as an archive for our papers, preprints, and ongoing work.

Finally, we have helped to organize international scientific meetings. In the past year, we have participated in organizing and running two such meetings plus one year–long program at the Fields Institute in Toronto. One meeting was a Workshop on Patterns in Physics, Nov. 14–18, 2003. The other is a Workshop on Free–Surface Water Waves, to be held June 14–18, 2004. Both are part of a Thematic Program on Partial Differential Equations, held at the Fields Institute during 2003–2004. For more information about either this program or the workshops, see:

`http://www.fields.utoronto.ca/programs/scientific/0304/pde/`

## 5 Papers Supported by this Grant


- Carter, J.D. & Segur, H., 2003, "Instabilities in the two–dimensional cubic nonlinear Schrödinger equation", *Phys Rev. E*, **68** 045601.
- Craig, W. & Nicholls, D., 2002, "Traveling gravity water waves in two and three dimensions", *Euro. J. Mech. B – Fluids,* **21**, pp. 615–641.
- Craig, W., Guyenne, P., & Kalish, H., 2004, "Hamiltonian long–wave expansions for free surfaces and interfaces", submitted for publication.
- Craig, W., Guyenne, P., Nicholls, D. & Sulem , C., 2004, "Hamiltonian long–wave expansions for water waves over a rough bottom", submitted for publication.
- Deconinck, B., Heil, M., Bobenko, A., van Hoeij, M. & Schmies, M., 2004, "Computing Riemann theta functions", *Math. of Computation*, **73**, pp. 1417–1442.
- Guyenne, P. & Nicholls, D., 2004, Numerical simulations of solitary waves on plane slopes", submitted for publication.
- Hammack, J.L. & Henderson, D.M., 2003, "Experiments on deep–water waves with two–dimensional surface patterns", *J. Offshore Mech. & Arctic Eng.,* **125**, pp. 48–53
- Hammack, J.L., Henderson, D.M. & Segur, H., 2004, "Deep–water waves with persistent, two–dimensional surface patterns", submitted for publication.
- Segur, H., Henderson, D., Carter, J.D., Hammack, J., Li, C.M., Pheiff, D. & Socha, K., 2004, "Stabilizing the Benjamin–Feir Instability", submitted for publication.
- D. Nicholls & F. Reitich, 2004, "On Analyticity of Traveling Water Waves," submitted for publication.


## 6 Other References


- Benjamin, T.B. & Feir, J.E., 1967, "The disintegration of wavetrains in deep water, Part 1", *J. Fluid Mech.*, **27**, pp. 417–430.
- Benney, D.J. & Newell, A.C., 1967, "The propagation of nonlinear wave envelopes", *Stud. App. Math.*, **46**, pp. 133–139.
- Cooker, M.J., Weidman, P.D. & Bale, D.S., 1997, "Reflection of a high–amplitude solitary waves at a vertical wall", *J. Fluid Mech.*, **342**, pp. 141–158.
- Craig, W. & Nicholls, D., 2000, "Traveling two and three dimensional capillary gravity waves", *SIAM: Math. Analysis,* **32**, pp. 323–359.
- Dysthe, K.B., 1979, "Note on a modification of the nonlinear Schrödinger equation for application to deep water waves", *Proc. Roy Soc. London A*, **369**, pp.105–114.
- Ioulalen, M. & Kharif, C., 1993, "Stability of three–dimensional progressive gravity waves on deep water to superharmonic disturbances", *Euro.J. Mech. B –Fluids*, **3**, pp. 401–414.

- Ioulalen, M. & Okamura, M., 2002, "Structure of the instability associated with harmonic resonances of short–crested waves", *J. Phys. Ocean.*, **32**, pp. 1331–1337.
- Lighthill, M.J., 1965, "Contribution to the theory of waves in nonlinear dispersive systems", *J. Inst. Math.Appl*, **1**, pp. 269–306.
- McLean, J.W., Ma, Y.C., Martin, D.U., Saffman, P.G. & Yuen, H.C, 1981, "Three dimensional instability of finite amplitude water waves", *Phys. Rev. Lett*, **46**, pp. 817–820.
- Miles, J.W., 1967, "Surface–wave damping in closed basins", *Proc. Roy. Soc. A*, **297**, pp. 459–475.
- Ostrovsky, L.A., 1967, "Propagation of wave packets and space–time self–focusing in a nonlinear medium," *Sov. Phys. JETP*, **24**, pp. 797–800.
- Reeder, J. & Shinbrot, M., 1981, "Three–dimensional nonlinear wave interaction in water of constant depth", *Nonlinear Analysis*, **5**, pp. 303–323.
- Rosales, R. & Papanicolaou, G., 1983, "Gravity waves in a channel with a rough bottom", *Stud. App. Math.*, **68**, pp. 89–102.
- Trulsen, K. & Dysthe, K.B., 1996, "A modified nonlinear Schrödinger equation for broader bandwidth gravity waves on deep water", *Wave Motion*, **24**, pp. 281–289.
- Whitham, G. B., 1967, "Nonlinear dispersion of water waves", *J. Fluid Mech.*, **27**, pp. 399–412.
- Wright, J. D., 2004, "Higher order corrections to the KdV approximation for water waves", Ph. D. Dissertation, Boston University.
- Zakharov, V.E., 1967, "Instability of self–focusing of light", *Sov. Phys. JETP*, **24**, pp. 455–459.
- Zakharov, V.E., 1968, "Stability of periodic waves of finite amplitude on the surface of a deep fluid", *J. Appl. Mech. Tech. Phys.*, **2**, pp. 190–194.