

2002

Qualifying Exam in Advanced Calculus (Retake)
Six questions - April 2002

1. Study the convergence of the series

$$1 - \frac{1}{2}x^2 + \frac{1 \cdot 3}{2 \cdot 4}x^4 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^6 + \dots$$

Determine the range of x for which convergence is assured.

2. Of what function $f(x, y, z)$ is

$$(y^2 + 2z^2x - 1) dx + 2yx dy + 2zx^2 dz$$

the differential? Show that the line integral of this differential is path-independent.

3. Find the moment of inertia

$$I = \iiint \rho(x^2 + y^2) dV$$

of a homogeneous right circular cone about its axis (taken as the z -axis). Let the radius of the top of the cone be b , the altitude be h and the density ρ be constant.

4. Find the shortest distance between the two lines in 3-space

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -4 \\ 5 \end{pmatrix} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} + \nu \begin{pmatrix} 8 \\ -3 \\ 1 \end{pmatrix} \quad -\infty < \mu, \nu < \infty$$

5. A particle moves in the x - y -plane according to the law

$$x(t) = 64\sqrt{3}t \quad y = 64t - 16t^2,$$

and is acted on by a force \mathbf{F} which is directly proportional (proportionality constant c) to the velocity in magnitude, but opposite in sense to the velocity. Find the work done by \mathbf{F} from $t = 0$ to $t = 4$.

6. Find the area of that part of the sphere $x^2 + y^2 + z^2 = a^2$ which is contained within the elliptic cylinder $4x^2 + y^2 = a^2$.

Hint: $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a}$.