

Preliminary Exam: Advanced Calculus

April 1, 2005

1. Find all positive values of x for which the infinite series

$$\sum_{s=1}^{\infty} s! \left(\frac{x}{s}\right)^s$$

converges.

2. A two-dimensional curve is given in its parametric form

$$x(t) = \frac{m}{\cos t} \quad y(t) = n(\tan t - t) \quad m, n > 0.$$

Determine all asymptotes of $y(x)$ as x tends to infinity.

3. Evaluate the definite integral

$$\int_0^{\infty} x^n e^{-ax} dx \quad (a > 0).$$

4. Determine the mass M and the coordinates (ξ, η) of the center of gravity of a two-dimensional curve S described by the cycloid

$$x(t) = r(t - \sin t) \quad y(t) = r(1 - \cos t) \quad (0 \leq t \leq 2\pi)$$

according to

$$M = \int_S ds \quad \xi = \frac{1}{M} \int_S x ds \quad \eta = \frac{1}{M} \int_S y ds.$$

5. Let K be a body of uniform density that is formed by the sphere $x^2 + y^2 + z^2 = 2az$ and the two conical surfaces $x^2 + y^2 = z^2 \tan^2 \alpha$ and $x^2 + y^2 = z^2 \tan^2 \beta$ with $\alpha < \beta$ (see Figure for a cross-sectional plot). Determine the volume of K .

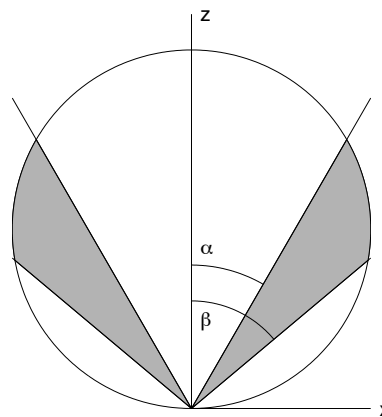


Figure 1: Sketch of geometry.