Advanced and Vector Calculus

Winter 2001

Vector Calculus

1. Find the area of the parallelogram with the following vertices:

$$(-4, 2),\ (-6, 5),\ (-3, 6),\ (-5, 9)\,.$$

2. In polar coordinates, the unit vectors are

$$\vec{e}_r = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}\,,$$

$$\vec{e}_\theta = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j}\,.$$

and a particle's position is specified as

$$\vec{r}(t) = r(t)\,\vec{e}_r\,.$$

Suppose a particle moves along a path with polar coordinates $r(t)$ and $\theta(t)$. What is its acceleration in polar coordinates?

3. Let

$$h = 9 - 2x^2 - \frac{1}{2}y^2 + 2x$$

denote the height on a mountain at position (x, y). In what direction from $(1, 1)$ is the steepest descent? What is your rate of change of elevation if you head northwest? Use your knowledge of the gradient to determine the top of the mountain.

4. Let S be the closed surface defined by the paraboloid

$$z = x^2 + y^2\,,$$

for $x^2 + y^2 \le 1$, along with the covering lid $x^2 + y^2 \le 1$, $z = 1$. The vector field $\vec{F}$ is

$$\vec{F} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}\,.$$

Verify the divergence theorem,

$$\iint_S \vec{F}\cdot\vec{n}\,dS = \iiint_V \vec{\nabla}\cdot\vec{F}\,dV\,,$$

for the surface S by doing both the volume and the surface integral. Feel free to choose a coordinate system that will simplify the integration.

Series and Such

5. Determine which of the following series converge and which diverge. Be sure to justify your answer.

(a) $S = \sum_{k=1}^{\infty} \frac{1}{k}$

(b) $S = \sum_{k=0}^{\infty} \frac{k}{2^k}$

6. Determine a power series representation of

$$f(x) = \frac{1}{2+x}\,,$$

valid about or in a neighborhood of $x = 1$. Determine the radius of convergence of this power series.

7. What is the difference between pointwise and uniform convergence to a function $f(x)$? Why do we care about this difference? What are some of the advantages or results that follow from uniform convergence?