2002 Winter 2002

Qualifying Exam in Advanced Calculus

1. Consider the infinite series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^s}$$

and determine for which value of s the series converges/diverges.

2. Determine the value of the infinite series

$$S = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \cdots$$

Hint: Relate the above series to the Taylor series expansion of an appropriately chosen function $f(x)$ about $x = 0$ evaluated at $x = 1$.

3. The area enclosed by the curve C is given as

$$A = \frac{1}{2} \int_C x\, dy - y\, dx.$$

Compute the area of the ellipse given by $(x/a)^2 + (y/b)^2 = 1$.

4. Determine the area of the surface $z = xy$ which is limited by the circle $x^2 + y^2 \leq R^2$.

5. Use Gauss' theorem to evaluate

$$\int\int_S \vec{F} \cdot d\vec{S}$$

with $\vec{F} = (x^3, y^3, z^3)$ and S as the sphere of radius R.

6. Calculate the second derivative of $f(x)$ for
   (a) $f(x) = x^x$,
   (b) $D(f(x), x) = 0$ with D a given function that is twice differentiable with respect to its arguments.

7. Find the curvature as a function of arclength for the right-handed helix of pitch $2b\pi$ defined by

$$\mathbf{r}(t) = a \cos \omega t\ \mathbf{i} + a \sin \omega t\ \mathbf{j} + bt\ \mathbf{k}$$

with $a, b, \omega > 0$ and $0 \leq t < \infty$.