

ODE Prelim Jan. 2001

(50 pts)

① Consider $y'' - 2y' - 8y = -40\cos 2t$ $y(0) = 2$ $y'(0) = 4$

- Find the homogeneous solution (don't apply initial conditions)
- Use undetermined coefficients to solve the nonhomogeneous problem.
- Use variation of parameters to solve the nonhomogeneous problem.
- Apply initial conditions to determine the unique solution
- Verify that by letting $x_1 = y$ and $x_2 = y'$ the equation can be written

$$\vec{x}' = \begin{pmatrix} 0 & 1 \\ 8 & 2 \end{pmatrix} \vec{x} - 40 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos 2t$$

- Use the systems approach to solve the homogeneous part and graph the phase-plane.
- Use variation of parameters to solve the nonhomogeneous problem
- Use undetermined coefficients to solve the nonhomogeneous problem
- Apply the initial conditions to determine the unique solution.

② (30 points) Consider the nonlinear system

$$x' = (2+x)(y-x) \quad \text{and} \quad y' = (4-x)(y+x)$$

Determine the equilibrium points and their stability (i.e. the eigenvectors and eigenvalues).

③ (20 points) Consider the second order equation

$$t^2 y'' - t(t+2)y' + (t+2)y = 0 \quad (t > 0)$$

Given a solution $y_1(t) = t$, find the second linearly independent solution y_2 .