ODE Prelim Jan. 2002

Winter 2002

① Consider the ODE with a, b, c arbitrary constants:

$$\frac{dy}{dx} = -\frac{ax+by}{bx+cy}$$

a) Show the equation is exact, and then solve using the exactness technique.

b) Solve by letting $z = y/x$

② Consider $y'' + 2y' + 5y = g(t) + 3\delta(t-\pi)$ $\quad y(0)=0, \; y'(0)=1$

$$g(t) = \begin{cases} 0 & 0 < t < 2\pi \\ \sin t & t \geq 2\pi \end{cases}$$

a) Express $g(t)$ in terms of a Heaviside function.

b) Solve the ODE using Laplace transforms.

③

Consider $4y'' - 4y' + y = 16e^{t/2}$

a) solve using undetermined coefficients.

b) write out the equation as a system of first order ODE's and solve using variation of parameters

④ Consider the nonlinear system $x' = x - y^2$ and $y' = y - x^2$

a) Determine the equilibrium points and their stability (i.e. the eigenvectors and eigenvalues).

b) sketch the phase plane

⑤ Consider $y'' + p(t)y' + q(t)y = 0$ $\quad y(0) = y_0, \; y'(0) = y_0'$

a) If y_1 and y_2 solve the equation, show that $c_1y_1 + c_2y_2$ is also a solution provided the Wronskian $W = y_1y_2' - y_1'y_2 \neq 0$

b) Derive an equation for the Wronskian that shows that it's either always zero or never zero.