Review of Logic

A proposition is a statement which is either true or false.

$p \Rightarrow q$: If p true then q true

e.g: $x>1 \Rightarrow x>0$ true

$x>0 \Rightarrow x>1$ false (counterexample $x=1/2$)

$p \Leftrightarrow q$: p true if and only if q

e.g. $2x+1=0 \Leftrightarrow 2x=-1$ true

$x^2=1 \Leftrightarrow x=1$ false (counterexample $x=-1$)

When you solve an equation, all steps must be true with "$\Leftrightarrow$":

$f(x)=g(x) \Leftrightarrow \cdots \Leftrightarrow x=a$ or $x=b$ or $x=c$.

If $p \Rightarrow q$ is true, then $q \Rightarrow p$ may or may not be true.

p

$p \vee q$: p true or q true or both true

e.g. $x^2 = 1 \Leftrightarrow x = 1 \vee x = -1$. true

$p \wedge q$: both p, q are true

e.g: $a^2 + b^2 = 0 \Rightarrow a = 0 \wedge b = 0$. true

$\bar{p}$: p is false.

e.g: $\overline{x=0} \Leftrightarrow x \neq 0$

$\overline{x>0} \Leftrightarrow x \leq 0$

Rules of Logic

(1) $\overline{p \wedge q} \Leftrightarrow \bar{p} \vee \bar{q}$ (2) $\overline{p \wedge q} \Leftrightarrow \bar{p} \vee \bar{q}$

~~(2)~~

(3) $(p \Rightarrow q) \Leftrightarrow (\bar{q} \Rightarrow \bar{p})$ (contrapositive)

e.g. the contrapositive of

$x > 1 \Rightarrow x > 2$

is $x \leq 2 \Rightarrow x \leq 1$.

Sets

A set A is a collection of objects (e.g. numbers, vectors, points, etc.), which we call elements)

$x \in A$: x belongs to A

$x \notin A$: x does not belong to A

$\emptyset$: empty set (has no elements).

For two sets A, B

$A \cap B$: all elements that belong to A and B

$$x \in A \cap B \Leftrightarrow x \in A \wedge x \in B$$

$A \cup B$: all elements that belong to A or B or both

$$x \in A \cup B \Leftrightarrow x \in A \vee x \in B$$

$\mathbb{R}$: set of all real numbers

$\mathbb{Q}$: set of all rational numbers
(can be written as ratio of integers)

$\mathbb{Z}$: set of all integers

$$\mathbb{Z} = \{0, \pm 1, \pm 2, \dots\}$$

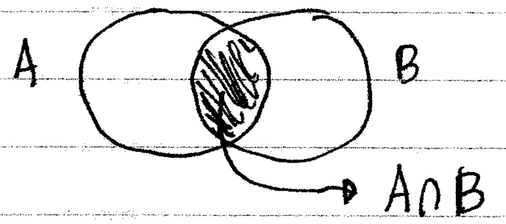
$\mathbb{N}$: set of all natural numbers

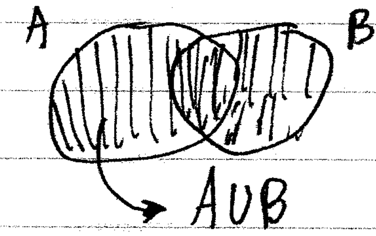
$$\mathbb{N} = \{0, 1, 2, \dots\}$$

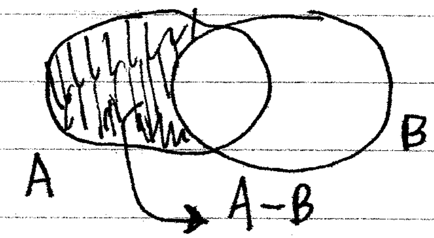
$A - B$: all elements of A not in B

$$x \in A - B \Leftrightarrow x \in A \wedge x \notin B$$

Venn Diagrams







Intervals

$[a,b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$

$(a,b) = \{x \in \mathbb{R} \mid a < x < b\}$

$(a,b] = \{x \in \mathbb{R} \mid a < x \leq b\}$

$[a,b) = \{x \in \mathbb{R} \mid a \leq x < b\}$

$(a,+\infty) = \{x \in \mathbb{R} \mid a < x\}$

$[a,+\infty) = \{x \in \mathbb{R} \mid a \leq x\}$

$(-\infty,a) = \{x \in \mathbb{R} \mid x < a\}$

$(-\infty,a] = \{x \in \mathbb{R} \mid x \leq a\}$

We use intervals to express solutions to equations or inequalities.

Quantifiers

$\forall x \in A : p(x)$ → The statement $p(x)$ is true for all $x \in A$.

$\exists x \in A : p(x)$ → There is at least one $x \in A$ such that the statement $p(x)$ is true.

example : For any non-zero number a, there is a number b such that $ab = 1$

$\forall a \in \mathbb{R} - \{0\} : \exists b \in \mathbb{R} : ab = 1$

example ; The definition of prime number.

$p \in \mathbb{N}$ prime number $\Leftrightarrow \forall q \in \mathbb{N} - \{1, p\}$: q does not divide p.

(1) The negation of $\forall x \in A : p(x)$ is $\exists x \in A : \bar{p}(x)$

(2) The negation of $\exists x \in A : p(x)$ is $\forall x \in A : \bar{p}(x)$

example : For $p, q \in \mathbb{N}$.

q divides $p \Leftrightarrow \exists r \in \mathbb{N} : p = qr$

q does not divide $p \Leftrightarrow \forall r \in \mathbb{N} : p \neq qr$.

p prime number $\Leftrightarrow \forall q \in \mathbb{N} - \{1, p\} : \forall r \in \mathbb{N} : p \neq qr$.