Solutions to Worksheet 8

Math 126 AA and AB

Problem 4

Part a

Set z = 16 in the second equation to get

$$16 = x^2 + 4y^2 \qquad \rightarrow \qquad 1 = \left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2$$

Parameterize this to get

$$r(t) = \langle 4\cos(t), 2\sin(t), 16\rangle$$

Part b

$$r'(t) = \langle -4\sin(t), 2\cos(t), 0\rangle$$

Part c

$$\int_0^{2\pi} \sqrt{16\sin^2(t) + 4\cos^2(t)}dt$$

Part d (corrected)

At the point $(2, \sqrt{3}, 16)$, $t = \frac{\pi}{3}$ (check this yourself). So, $r'(\frac{\pi}{3})$ is given by

$$r'(\frac{\pi}{3}) = \left\langle -2\sqrt{3}, 1, 0 \right\rangle$$

So, this gives that the equation of the tangent at $(2, \sqrt{3}, 16)$ is

$$\begin{aligned} x &= -2\sqrt{3}t + 2 \\ y &= t + \sqrt{3} \\ z &= 16 \end{aligned}$$

Part e

Because the plane is parallel to the x-axis, the direction vector $\langle 1, 0, 0\rangle$ lies in the plane. Additionally, if the plane contains the points $(0, 0, 3)$ and $(0, 5, 0)$, then it also contains the vector $\vec{AB} = \langle 0, 5, -3\rangle$. Therefore, the normal vector for the plane is the cross product of the two vector that lie in the plane $n = \langle 0, 3, 5\rangle$. The equation of the plane then becomes

$$0 \cdot (x - 0) + 3 \cdot (y - 0) + 5 \cdot (z - 3) = 0 \quad \rightarrow \quad 3y + 5z = 15$$

Part f (corrected)

For this problem, plug in the parametric equations for the line into the equation for the plane to get

$$3y + 5z = 15 \quad \rightarrow \quad 3(t + \sqrt{3}) + 5(16) = 15$$

Now all you have to do is solve for t to get

$$t = -\frac{65}{3} - \sqrt{3} \quad \rightarrow \quad (x, y, z) = \left(-2\sqrt{3}\left(-\frac{65}{3} - \sqrt{3}\right) + 2, -\frac{65}{3}, 16\right)$$

YUCK!

Problem 5

You know that in order to use everybody's favorite trig identity $\sin^2(t) + \cos^2(t) = 1$, you need to add 5 to the x component so that you get something like $(\cos(t) + 5 - 5)^2 = \cos^2(t)$. Now, if we do this and stick with the standard equation of the circle, we will end up starting at the point $(6, 0)$, but if you change the $cos(t) \rightarrow -cos(t)$, the problem is solved.

$$x(t) = 5 - \cos(t) \quad y(t) = \sin(t)$$

Problem 6

Part a

Recall that

$$\kappa(t) = \frac{\left|r'(t) \times r''(t)\right|}{\left|r'(t)\right|^3}$$

If you calculate this, you get that $\kappa(t) = \frac{4}{25}$.

Part b

$$\frac{ds}{dt} = \left|r'(t)\right| = 5$$

This means, that if we start our parameterization at $t = 0$, $s(t) = 5t \rightarrow t = \frac{s}{5}$. Thus,

$$r(s) = \left\langle 4\cos\left(\frac{s}{5}\right), 4\sin\left(\frac{s}{5}\right), \frac{3s}{5}\right\rangle$$

Part c

The curvature doesn't depend on the parameterization, so we know that

$$\kappa(t) = \frac{\left|q'(t) \times q''(t)\right|}{\left|q'(t)\right|^3} = \frac{4}{25}$$

So, all we have to calculate is $\left|q'(t)\right|^3 = (15t^2)^3$. Thus $\left|q'(t) \times q''(t)\right| = \frac{4}{25} \cdot (15t^2)^3 = 540t^6$