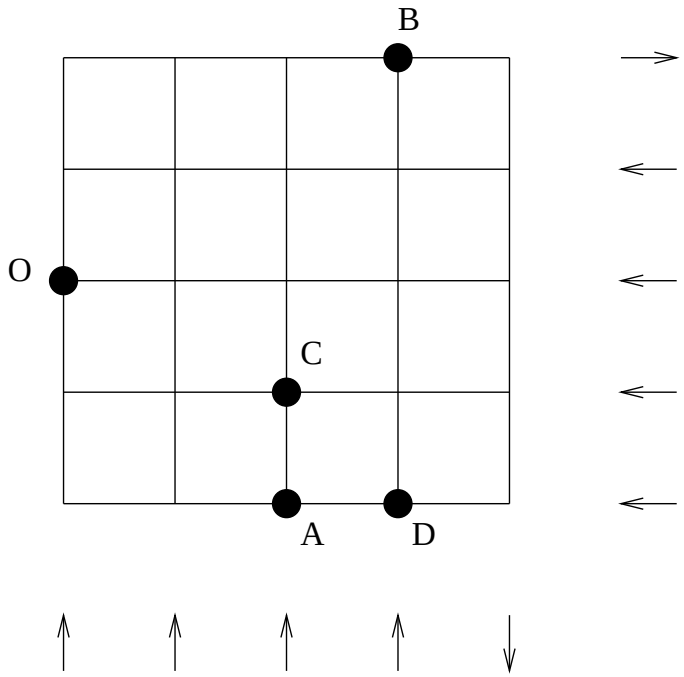
Network Flow Models

Homework due Friday, October 8:

1. Below is a map of Oneway City, a poorly planned town with one-way streets as indicated. A number of locations are also indicated on the map.



   (a) Draw the complete directed graph with vertices corresponding to O, A, B, and C (ignore D for now) and two edges between each pair of vertices, one in each direction. Label each with the distance (number of blocks) one must travel to get from one location to the other. For example, the distance from A to C is 1 block and from C to A is 11 blocks.

   (b) Suppose a delivery van leaving from O must visit A, B, C, and D and return to O. Use the branch and bound algorithm to find the best route. Show the portion of the tree that you constructed in finding this route, indicating the partial distances computed and indicating what parts of the tree you were able to prune off. (It is pretty obvious what the best route is in this case; I constructed it to make the branch and bound algorithm fairly simple to apply. You might want to experiment with similar problems where the answer isn't so obvious.)

2. Do problems 1(b) (using Dijkstra's algorithm) and 2 on the page of problems from Hillier and Lieberman, *Operations Research*.

3. An upper-triangular linear system has the form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

   for example, in the case of a 4×4 system. Such systems are easily solved by *back substitution* (solve the last equation for x_4, then the previous equation for x_3, etc.)

   How many multiplication operations does this require to accomplish? (Count a division the same as a multiplication, so computing $x_4 = b_4/a_{44}$ takes 1 "multiply", for example.)

   Exactly how many "multiplies" does it take to solve a general $n \times n$ upper-triangular system? In particular, how many to solve an upper-triangular system of 1000 equations?

Homework due Monday, October 11:

Write a description of Dijkstra's algorithm for finding the shortest path between two given nodes in a network. Design this to be suitable for students at your level, or other people with similar background, who have not taken a course in graph theory algorithms (or this class). Explain what the goal of the algorithm is, why it is important (perhaps in the context of a real problem), and how it works. Explain it in such a way that someone reading only your document, with no other books or lectures on the subject, could figure out the algorithm and apply it to solve a problem. You will probably want to use illustrations and a specific example in order to make it clear, but try also to be fairly concise (2 or 3 pages, at most).

Write these up neatly enough that others can read them. Monday you will exchange papers and have a chance to comment on another person's document, so please make it easy on each other!