Math 381 R. J. LeVeque
Handout #8 November 3, 1999

Linear Programming Homework

Read the class notes on setting up linear programming problems. The next few lectures we will look at some examples. Please take a look at Model 3 and Model 11 on the webpage

`http://www.math.washington.edu/~burke/crs/407/models/`

and think about how you might set these up. You don't need to turn anything in.

Due Wednesday, November 10:

1. Recall the problem of life vests and life boats from Chapter 1 of the class notes. For simplicity consider smaller boats with the parameters

$$\begin{aligned} C_1 &= 1 \quad \text{(capacity of each vest)} \\ C_2 &= 2 \quad \text{(capacity of each boat)} \\ C &= 10 \quad \text{(total capacity needed)} \\ V_1 &= 1 \quad \text{(volume of each vest)} \\ V_2 &= 5 \quad \text{(volume of each boat)} \\ V &= 15 \quad \text{(total volume available)} \end{aligned}$$

   (a) Set up the problem of maximizing the number of life boats that can be carried as a linear programming problem. **To begin with, ignore the constraints that x_1 and x_2 should be integers. Also, forget about the constraint that x_2 should be an even number.**

   (b) Use the "graphical method" to find the solution to this problem (ignoring the integer constraints). Draw the feasible region in the x_1-x_2 plane and draw contour lines of the objective function, indicating the corner of the feasible set at which the optimum occurs. Solve the appropriate linear system to find this point.

   (c) Now introduce the constraint that x_1 and x_2 must be integers. This means only a finite number of discrete points from the previous feasible set are still feasible (points for which x_1 and x_2 are both intergers). Find all feasible points and plot them. Use the graphical method to solve this integer programming problem.

   (d) Suppose the life boats are smaller and only require volume $V_2 = 2.5$. Repeat part (b) for this case.

2. Mariko has many interests and wants to take a number of crafts and sports classes during the summer at the community center. In fact there are 7 different course she would like to take. But each one requires 5 hours per week of commitment and costs \$15/week. She has a job which pays \$10/hour, and she must work enough to earn \$200/week plus the cost of any classes she takes. Her boss has told her that she can work any number of hours she chooses between 25 and 40 each week. She is willing to spend at most 55 hours a week between her job and any classes she takes. She wants to determine how many classes she can take.

   (a) Let x_1 represent the number of hours per week she works, and x_2 the number of hours per week she spends on classes. First suppose that both of these can take any value (ignoring the fact that x_2 must be a multiple of 5). Set up her problem as trying to maximize x_2 subject to some constraints. Write out all of the constraints mathematically and draw the feasible set in the x_1-x_2 plane. Indicate where the solution to the linear programming problem is.

   (b) Determine this solution (x_1, x_2) explicitly by setting up and solving the appropriate 2×2 linear system. Write down and this system, don't solve the problem using linear programming software (though you might want to do so to check your solution).

   (c) Now solve the actual problem to determine the number of classes she can take, with the additional constraint on x_2 (but allowing x_1 to take any value). Is there a unique solution? How might she decide between the possible solutions?

   Note: This is called a *mixed integer programming problem* since it has a mixture of variable types, some of which must be integer, and the others continuous.

3. Consider Model 20 from
   `http://www.math.washington.edu/~burke/crs/407/models/`

   (a) Set this up as a linear programming problem.

   (b) Solve this using MATLAB, LINDO, or other linear programming software. (Note: In MATLAB, you can specify the upper bounds as `vub = [Inf; Inf; 20]`, where `Inf` means infinity in matlab.)

   (c) Which of the constraints are "binding" at the optimimum? (E.g. which of the three machines are being used at maximum capacity and which are not?)